I113 オートマトンと形式言語
レポート1の解説
(I113 Automaton & Formal Languages Answer & Comments for Report 1)

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レポート (1)

[問題1] 関数 f_i を以下のように定義する。

$$
\begin{cases}
f_0=0 \\
f_1=1 \\
f_i= f_{i-1}+f_{i-2} \text{ ただし } i>1
\end{cases}
$$

この数列(0,1,1,2,3,5,8,…)はフィボナッチ数列と呼ばれている。個々の数をフィボナッチ数と呼ぶ。このとき、以下の主張は明らかに間違っている。どこが間違っているかを示せ。

Report (1)

[Problem 1] Define a function f_i as follows:

$$\begin{cases} f_0=0 \\ f_1=1 \\ f_i = f_{i-1}+f_{i-2}, \text{ where } i>1. \end{cases}$$

This sequence (0,1,1,2,3,5,8,･･･) is called Fibonacci sequence. Each number is called Fibonacci number.

Then, the following claim is clearly wrong.

Point the error in the proof.

レポート (1)

$$\begin{cases} f_0 = 0 \\ f_1 = 1 \\ f_i = f_{i-1} + f_{i-2} \quad \text{ただし } i > 1 \end{cases}$$

[主張] 任意のフィボナッチ数の値は0である。

[証明] フィボナッチ数 f_i の i に関する帰納法で示す。

まず i =0 のとき、数列の定義から f_0=0 なので主張が成立する。

次に i >0 のときを考える。任意の $i' \leqq i$ に関して主張が成立すると仮定して、f_{i+1}の値を考える。定義より、$f_{i+1}=f_i+f_{i-1}$である。帰納法の仮定から f_i=0, f_{i-1}=0 なので、f_{i+1}=0+0=0となる。したがって主張は成立する。

Report (1)

$$\begin{cases} f_0 = 0 \\ f_1 = 1 \\ f_i = f_{i-1} + f_{i-2}, \text{ where } i > 1 \end{cases}$$

[Claim] Any Fibonacci number is equal to 0.

[Proof] Induction for the index i of Fibonacci number f_i.

When i =0, by the definition of the sequence, f_0=0. Hence we have the claim.

Suppose i >0. We assume that the claim holds for any $i' \leqq i$, and consider for f_{i+1}. By definition, $f_{i+1}=f_i+f_{i-1}$. On the other hand, by inductive hypothesis, we have f_i=0 and f_{i-1}=0. Thus f_{i+1}=0+0=0, which completes the proof.

レポート (1)

[帰納法の正しさ]

1. [基礎ステップ]が論理的に正しい
2. [帰納ステップ]の前提条件が正しい
3. [帰納ステップ]が論理的に正しい

Report (1)

[Correctness of Induction]

1. [Basic step] is correct.
2. [Inductive step] has correct hypothesis.
3. [Inductive step] is correct.

[帰納法の正しさ]
[基礎ステップ]が論理的に正しい
[帰納ステップ]の前提条件が正しい
[帰納ステップ]が論理的に正しい

レポート (1)

$$\begin{cases} f_0 = 0 \\ f_1 = 1 \\ f_i = f_{i-1} + f_{i-2} \quad ただし\ i > 1 \end{cases}$$

[主張] 任意のフィボナッチ数の値は0である。

[証明] フィボナッチ数 f_i の i に関する帰納法で示す。

まず $i=0$ のとき、数列の定義から $f_0=0$ なので主張が成立する。

次に $i>0$ のときを考える。任意の $i' \leqq i$ に関して主張が成立すると仮定して、f_{i+1} の値を考える。定義より、$f_{i+1}=f_i+f_{i-1}$ である。帰納法の仮定から $f_i=0$, $f_{i-1}=0$ なので、$f_{i+1}=0+0=0$ となる。したがって主張は成立する。

[基礎]ステップは最初の二つの場合の正しさを示さないとダメ

f_{i+1} に対する主張の正しさは、f_i, f_{i-1} に対する主張の正しさに依存している。

定義より、$f_0=0$ だが $f_1=1$ なので、主張は成立しない。

Report (1)

[Correctness of Induction]
[Basic step] is correct.
[Inductive step] has correct hypothesis.
[Inductive step] is correct.

$$\begin{cases} f_0 = 0 \\ f_1 = 1 \\ f_i = f_{i-1} + f_{i-2}, \text{ where } i > 1 \end{cases}$$

[Claim] Any Fibonacci number is equal to 0.

[Proof] Induction for the index i of Fibonacci number f_i.

When $i = 0$, by the definition of the sequence, $f_0 = 0$. Hence we have the claim.

Suppose $i > 0$. We assume that the claim holds for any $i' \leqq i$, and consider for f_{i+1}. By definition, $f_{i+1} = f_i + f_{i-1}$. On the other hand, by inductive hypothesis, we have $f_i = 0$ and $f_{i-1} = 0$. Thus $f_{i+1} = 0 + 0 = 0$, which completes the proof.

[Basic] step needs to show the correctness of the first two cases.

By definition, $f_0=0$, but $f_1=1$. Hence the claim does not hold.

The correctness of the claim for f_{i+1} stands for the correctness of two claims for f_i, and f_{i-1}.

[帰納法の正しさ]
[基礎ステップ]が論理的に正しい
[帰納ステップ]の前提条件が正しい
[帰納ステップ]が論理的に正しい

レポート (1)

$$\begin{cases} f_0 = 0 \\ f_1 = 1 \\ f_i = f_{i-1} + f_{i-2} \end{cases}$$ ただし $i > 1$

[解答例] 帰納ステップにおいて、f_{i+1}の場合の正当性を示すときに、f_i と f_{i-1} の正当性を使用している。

したがって、基礎ステップでは f_0 と f_1 の正当性を示す必要があるが、証明中では f_0 の正当性しか示していない。

実際、定義より f_1=1 なので、この時点で主張は成立していない。

[Correctness of Induction]
[Basic step] is correct.
[Inductive step] has correct hypothesis.
[Inductive step] is correct.

Report (1)

$$\begin{cases} f_0 = 0 \\ f_1 = 1 \\ f_i = f_{i-1} + f_{i-2}, \text{ where } i > 1 \end{cases}$$

[Answer] To show the correctness of the claim for f_{i+1} on the inductive step, the proof uses the correctness of the claim for two cases f_i and f_{i-1}.

Hence, it is necessary to show the correctness of the claim for f_0 and f_1 in the basic step. However, in the proof, only the correctness for f_0 is proved.

In fact, by definition, we have f_1=1 and then the claim is not true for i =1.

レポート(1)

[問題2] ε-NFAでは、初期状態は1つであるが、受理状態の個数について制限はない。また、受理状態からの遷移もありえる。しかしここで「受理状態は一つである」「受理状態からの遷移はない」という制限を加えても一般性を失わない。それはなぜか、を形式的に示せ。

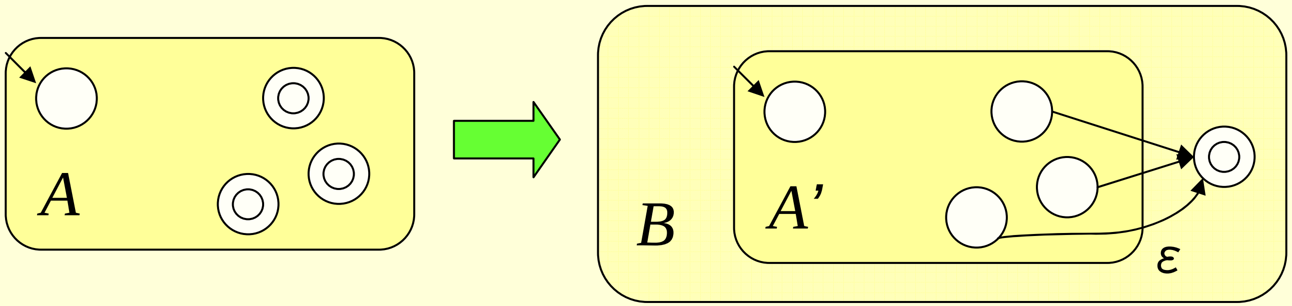
Report (1)

[Problem 2] While an ε-NFA has one initial state, there are no limits for the number of accepting states. Moreover, there can be a transition from an accepting state. However, we can assume that 'there is one accepting state' and 'there is no transitions from the accepting state' without loss of generality. Prove it why.

[問題2] ε-NFAに「受理状態は一つである」「受理状態からの遷移はない」という制限を加えても一般性を失わない。それはなぜか、を形式的に示せ。

[形式的でない説明]

与えられた ε-NFA *A* と同じ言語を受理する、条件を満たす ε-NFA *B* を以下のように構成する:

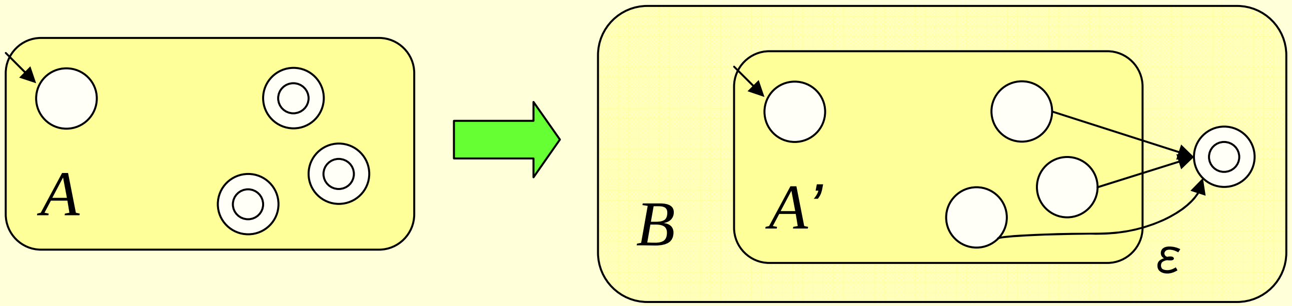


★これは直感的な説明であって、形式的に示したことにはならない。

[Problem 2] We can assume that an ε-NFA satisfies 'there is one accepting state' and 'there is no transitions from the accepting state' without loss of generality. Prove it why.

[Informal description]

Given ε-NFA *A*, we can construct ε-NFA *B* that satisfy *L(A)=L(B)* and the conditions:



★This is just 'description', not a formal proof.

[問題2] ε−NFAに「受理状態は一つである」「受理状態からの遷移はない」という制限を加えても一般性を失わない。それはなぜか、を形式的に示せ。

[解答例]

与えられた ε−NFA $A = \{Q, \Sigma, \delta, q, F\}$ に対し、ε−NFA $B = \{Q', \Sigma, \delta', q, F'\}$を以下のように定義する:

$Q' := Q \cup \{q'\}$

$F' := \{q'\}$

$$\delta(p,x) = \begin{cases} \delta(p,x) & \text{すべての } p \in Q, x \in \Sigma \bigcup \{\varepsilon\} \\ q' & p \in F, x = \varepsilon \end{cases}$$

このとき $L(A)=L(B)$ で B は2つの制限を満たす。

[Problem 2] We can assume that an ε-NFA satisfies 'there is one accepting state' and 'there is no transitions from the accepting state' without loss of generality. Prove it why.

[Solution]

For given ε-NFA $A = \{Q, \Sigma, \delta, q, F\}$, define an ε-NFA $B = \{Q', \Sigma, \delta', q, F'\}$ as follows:

$$Q' := Q \cup \{q'\}$$

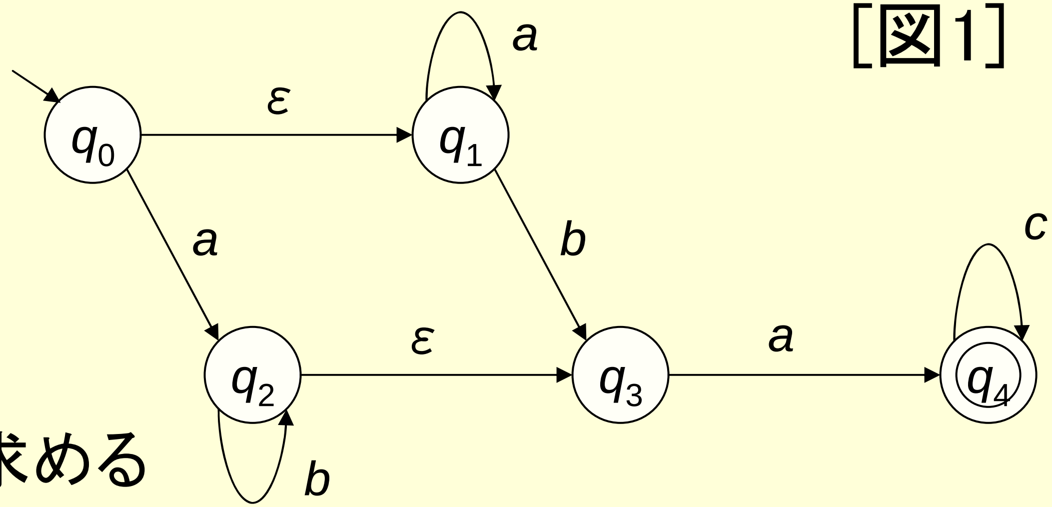
$$F' := \{q'\}$$

$$\delta(p, x) = \begin{cases} \delta(p, x) & \text{for all } p \in Q, x \in \Sigma \bigcup \{\varepsilon\} \\ q' & p \in F, x = \varepsilon \end{cases}$$

Then $L(A)=L(B)$ and B satisfies two conditions.

［問題3］ 図1で与えられるε−NFA $A=(\{q_0,q_1,q_2,q_3,q_4\}, \{a,b,c\}, \delta, q_0, \{q_4\})$が受理する言語と同じ言語を受理するDFAを構成せよ。

［図1］



[構成1] *ECLOSE*を求める

$$ECLOSE(q_0)=\{q_0,q_1\}$$

$$ECLOSE(q_1)=\{q_1\}$$

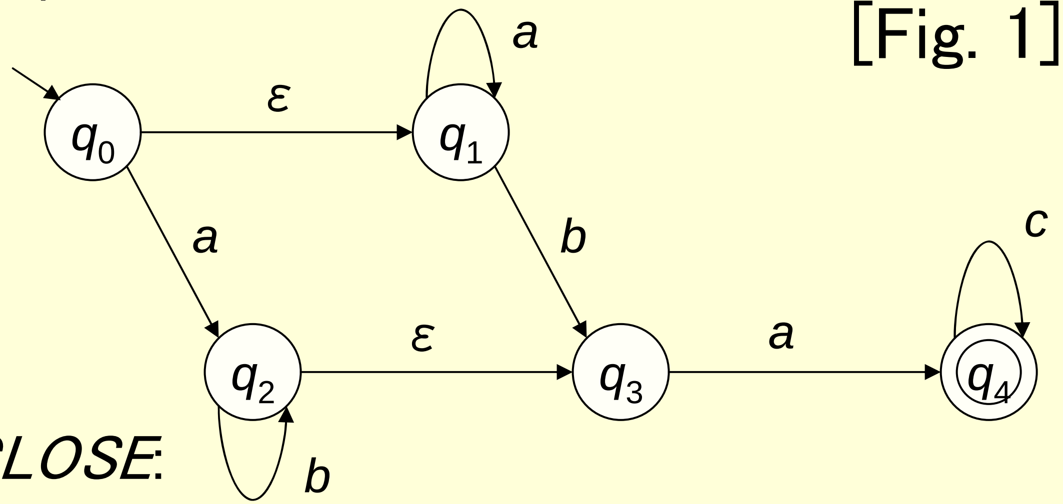
$$ECLOSE(q_2)=\{q_2,q_3\}$$

$$ECLOSE(q_3)=\{q_3\}$$

$$ECLOSE(q_4)=\{q_4\}$$

[Problem 3] Construct a DFA that accepts the same language accepted by an ε-NFA $A=(\{q_0,q_1,q_2,q_3,q_4\}, \{a,b,c\}, \delta, q_0, \{q_4\})$ given in Figure 1.

[Fig. 1]



[Construction 1] *ECLOSE*:

$$ECLOSE(q_0) = \{q_0, q_1\}$$

$$ECLOSE(q_1) = \{q_1\}$$

$$ECLOSE(q_2) = \{q_2, q_3\}$$

$$ECLOSE(q_3) = \{q_3\}$$

$$ECLOSE(q_4) = \{q_4\}$$

[構成1] *ECLOSE*

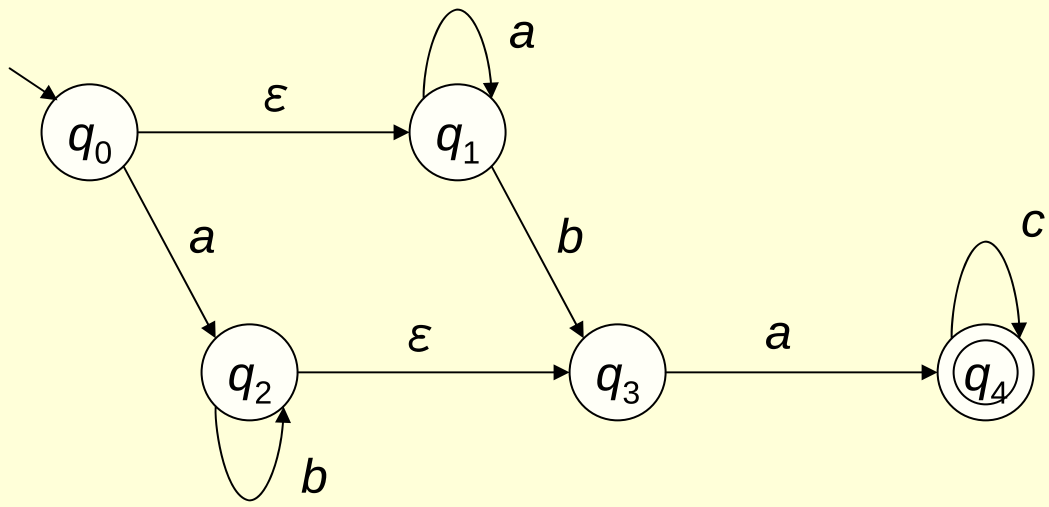
$$ECLOSE(q_0) = \{q_0, q_1\}$$

$$ECLOSE(q_1) = \{q_1\}$$

$$ECLOSE(q_2) = \{q_2, q_3\}$$

$$ECLOSE(q_3) = \{q_3\}$$

$$ECLOSE(q_4) = \{q_4\}$$



[構成2] 構成するDFAを $B = (Q, \{a,b,c\}, \delta', q_0', F\}$とおくと、

$$q_0' = ECLOSE(q_0) = \{q_0, q_1\}$$

$$\delta'(q_0', a) = ECLOSE(\delta(q_0, a)) \cup ECLOSE(\delta(q_1, a))$$

$$= ECLOSE(q_2) \cup ECLOSE(q_1) = \{q_1, q_2, q_3\}$$

q_1'

$$\delta'(q_0', b) = ECLOSE(\delta(q_0, b)) \cup ECLOSE(\delta(q_1, b)) = \{q_3\}$$

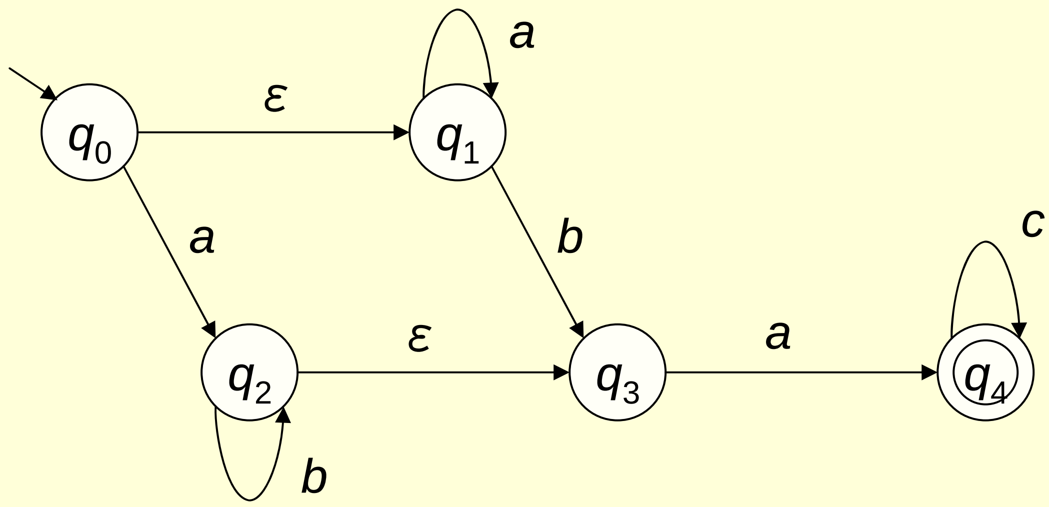
q_2'

$$\delta'(q_0', c) = ECLOSE(\delta(q_0, c)) \cup ECLOSE(\delta(q_1, c)) = \{\} = \phi$$

q_3'

[Construction 1] *ECLOSE*

$$ECLOSE(q_0) = \{q_0, q_1\}$$
$$ECLOSE(q_1) = \{q_1\}$$
$$ECLOSE(q_2) = \{q_2, q_3\}$$
$$ECLOSE(q_3) = \{q_3\}$$
$$ECLOSE(q_4) = \{q_4\}$$



[Construction 2] Let $B = (Q, \{a,b,c\}, \delta', q_0', F\}$ be the DFA. Then

$q_0' = ECLOSE(q_0) = \{q_0, q_1\}$

$\delta'(q_0', a) = ECLOSE(\delta(q_0, a)) \cup ECLOSE(\delta(q_1, a))$

$= ECLOSE(q_2) \cup ECLOSE(q_1) = \{q_1, q_2, q_3\}$ — q_1'

$\delta'(q_0', b) = ECLOSE(\delta(q_0, b)) \cup ECLOSE(\delta(q_1, b)) = \{q_3\}$ — q_2'

$\delta'(q_0', c) = ECLOSE(\delta(q_0, c)) \cup ECLOSE(\delta(q_1, c)) = \{\} = \phi$ — q_3'

[構成1] *ECLOSE*

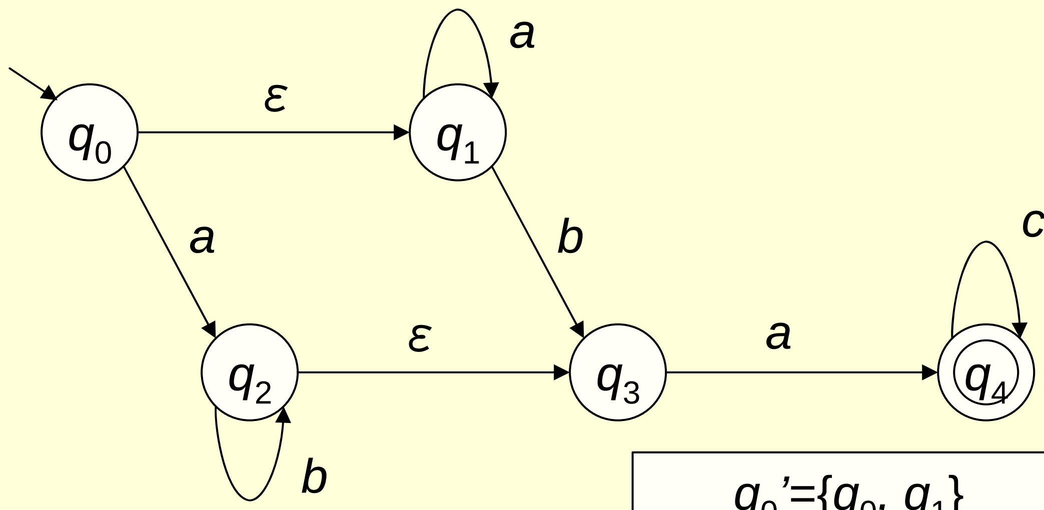
$$ECLOSE(q_0) = \{q_0, q_1\}$$

$$ECLOSE(q_1) = \{q_1\}$$

$$ECLOSE(q_2) = \{q_2, q_3\}$$

$$ECLOSE(q_3) = \{q_3\}$$

$$ECLOSE(q_4) = \{q_4\}$$



$q_0' = \{q_0, q_1\}$
$q_1' = \{ q_1, q_2, q_3 \}$
$q_2' = \{q_3\}$
$q_3' = \phi$

[構成2] 構成するDFAを $B = (Q, \{a,b,c\}, \delta', q_0', F\}$と

$q_1' = \{ q_1, q_2, q_3 \}$

$\delta'(q_1', a) = \{ q_1, q_4 \}$ ← q_4'

$\delta'(q_1', b) = \{ q_2, q_3 \}$ ← q_5'

$\delta'(q_1', c) = \{\} = \phi$

$q_2' = \{ q_3 \}$

$\delta'(q_2', a) = \{ q_4 \}$ ← q_6'

$\delta'(q_2', b) = \{\} = \phi$

$\delta'(q_2', c) = \{\} = \phi$

$q_3' = \phi$

$$\delta'(q_3', a) = \delta'(q_3', b) = \delta'(q_3', c) = \{\} = \phi$$

[Construction 1] *ECLOSE*

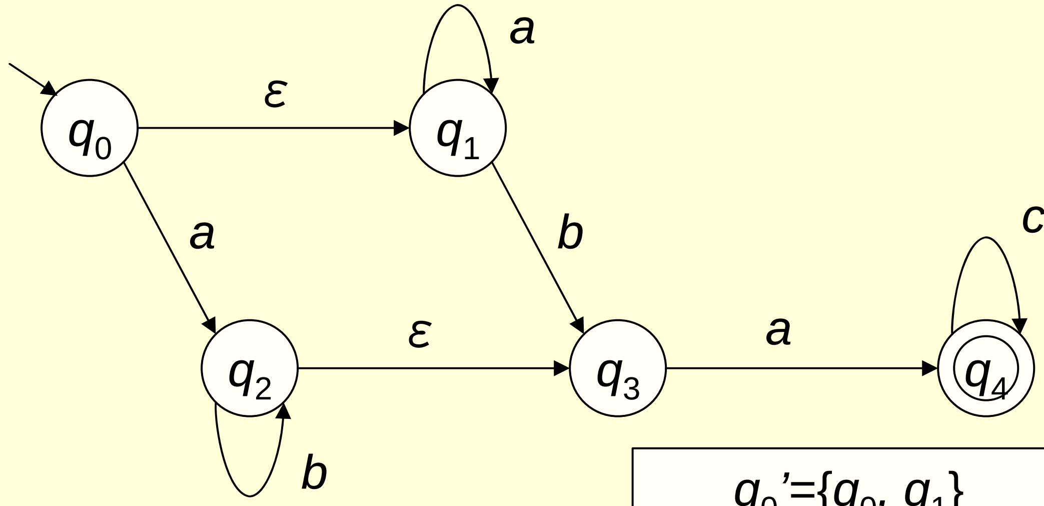
$$ECLOSE(q_0) = \{q_0, q_1\}$$

$$ECLOSE(q_1) = \{q_1\}$$

$$ECLOSE(q_2) = \{q_2, q_3\}$$

$$ECLOSE(q_3) = \{q_3\}$$

$$ECLOSE(q_4) = \{q_4\}$$



$q_0' = \{q_0, q_1\}$
$q_1' = \{ q_1, q_2, q_3 \}$
$q_2' = \{q_3\}$
$q_3' = \phi$

[Construction 2] Let $B = (Q, \{a,b,c\}, \delta', q_0', F\}$ be th

$q_1' = \{ q_1, q_2, q_3 \}$

$\delta'(q_1', a) = \{ q_1, q_4 \}$ → q_4'

$\delta'(q_1', b) = \{ q_2, q_3 \}$ → q_5'

$\delta'(q_1', c) = \{\} = \phi$

$q_2' = \{ q_3 \}$

$\delta'(q_2', a) = \{ q_4 \}$ → q_6'

$\delta'(q_2', b) = \{\} = \phi$

$\delta'(q_2', c) = \{\} = \phi$

$q_3' = \phi$

$$\delta'(q_3', a) = \delta'(q_3', b) = \delta'(q_3', c) = \{\} = \phi$$

[構成1] *ECLOSE*

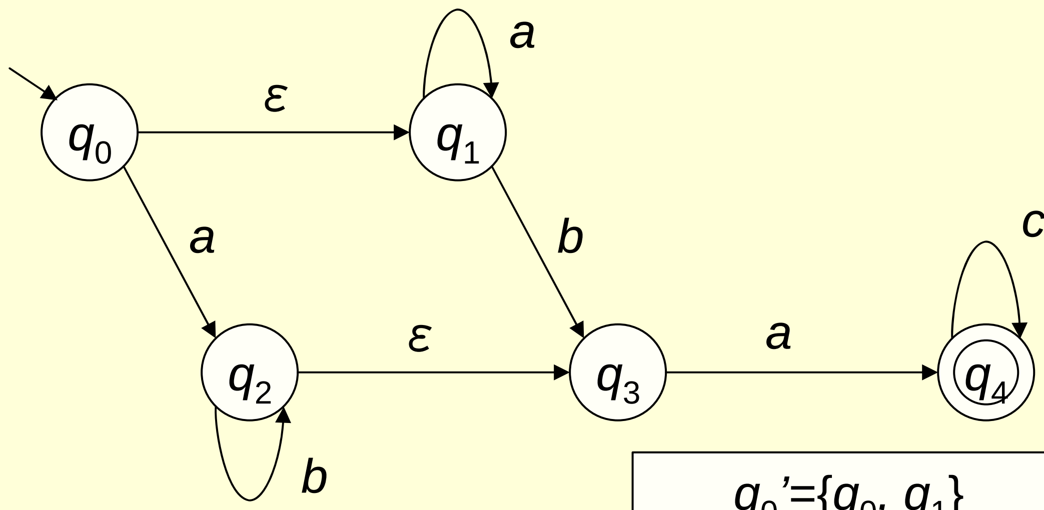
$$ECLOSE(q_0) = \{q_0, q_1\}$$

$$ECLOSE(q_1) = \{q_1\}$$

$$ECLOSE(q_2) = \{q_2, q_3\}$$

$$ECLOSE(q_3) = \{q_3\}$$

$$ECLOSE(q_4) = \{q_4\}$$



$q_0' = \{q_0, q_1\}$
$q_1' = \{q_1, q_2, q_3\}$
$q_2' = \{q_3\}$
$q_3' = \phi$
$q_4' = \{q_1, q_4\}$
$q_5' = \{q_2, q_3\}$
$q_6' = \{q_4\}$

[構成2] 構成するDFAを $B = (Q, \{a,b,c\}, \delta', q_0', F\}$とお

$\delta'(q_4', a) = \{q_1\}$ ← q_7'

$\delta'(q_4', b) = \{q_3\}$

$\delta'(q_4', c) = \{q_4\}$

$\delta'(q_6', a) = \phi$

$\delta'(q_6', b) = \phi$

$\delta'(q_6', c) = \{q_4\}$

$\delta'(q_5', a) = \{q_4\}$

$\delta'(q_5', b) = \{q_2, q_3\}$

$\delta'(q_5', c) = \phi$

$\delta'(q_7', a) = \{q_1\}$

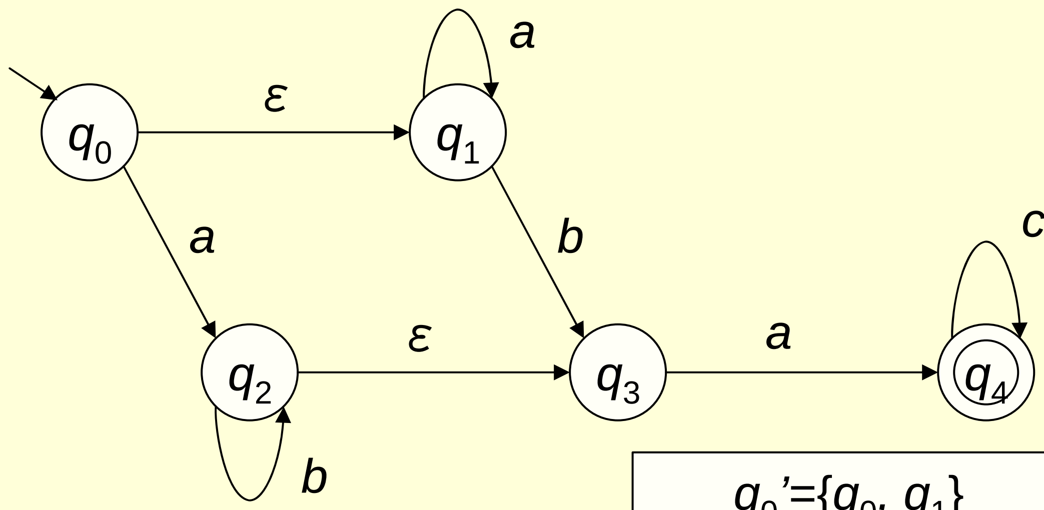
$\delta'(q_7', b) = \{q_3\}$

$\delta'(q_7', c) = \phi$

$F = \{q_4', q_6'\}$

[Construction 1] *ECLOSE*

$$ECLOSE(q_0) = \{q_0, q_1\}$$
$$ECLOSE(q_1) = \{q_1\}$$
$$ECLOSE(q_2) = \{q_2, q_3\}$$
$$ECLOSE(q_3) = \{q_3\}$$
$$ECLOSE(q_4) = \{q_4\}$$



$q_0' = \{q_0, q_1\}$
$q_1' = \{ q_1, q_2, q_3 \}$
$q_2' = \{ q_3 \}$
$q_3' = \phi$
$q_4' = \{ q_1, q_4 \}$
$q_5' = \{ q_2, q_3 \}$
$q_6' = \{ q_4 \}$

[Construction 2] Let $B = (Q, \{a,b,c\}, \delta', q_0', F\}$ be the

$\delta'(q_4', a) = \{ q_1 \}$ — q_7'

$\delta'(q_4', b) = \{ q_3 \}$

$\delta'(q_4', c) = \{ q_4 \}$

$\delta'(q_6', a) = \phi$

$\delta'(q_6', b) = \phi$

$\delta'(q_6', c) = \{ q_4 \}$

$\delta'(q_5', a) = \{ q_4 \}$

$\delta'(q_5', b) = \{ q_2, q_3 \}$

$\delta'(q_5', c) = \phi$

$\delta'(q_7', a) = \{ q_1 \}$

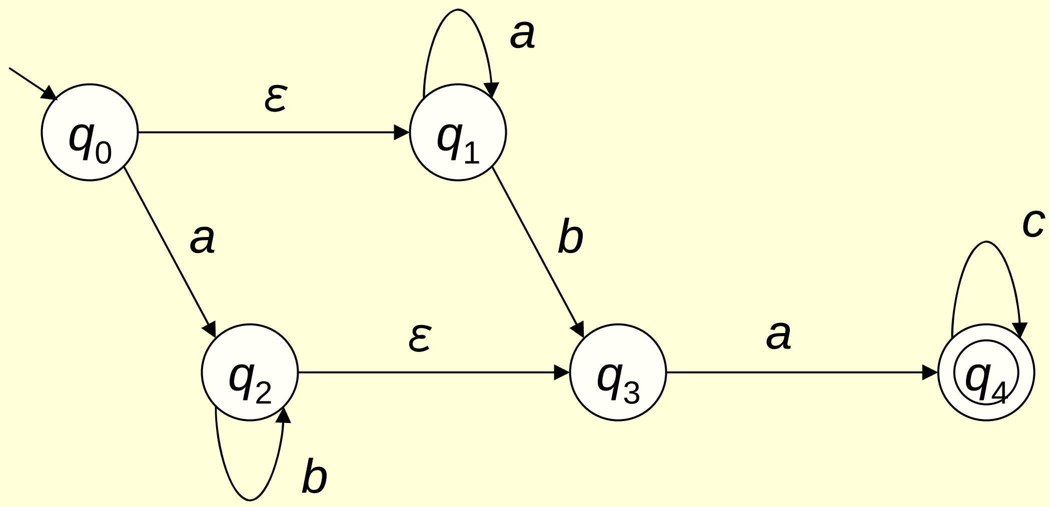
$\delta'(q_7', b) = \{ q_3 \}$

$\delta'(q_7', c) = \phi$

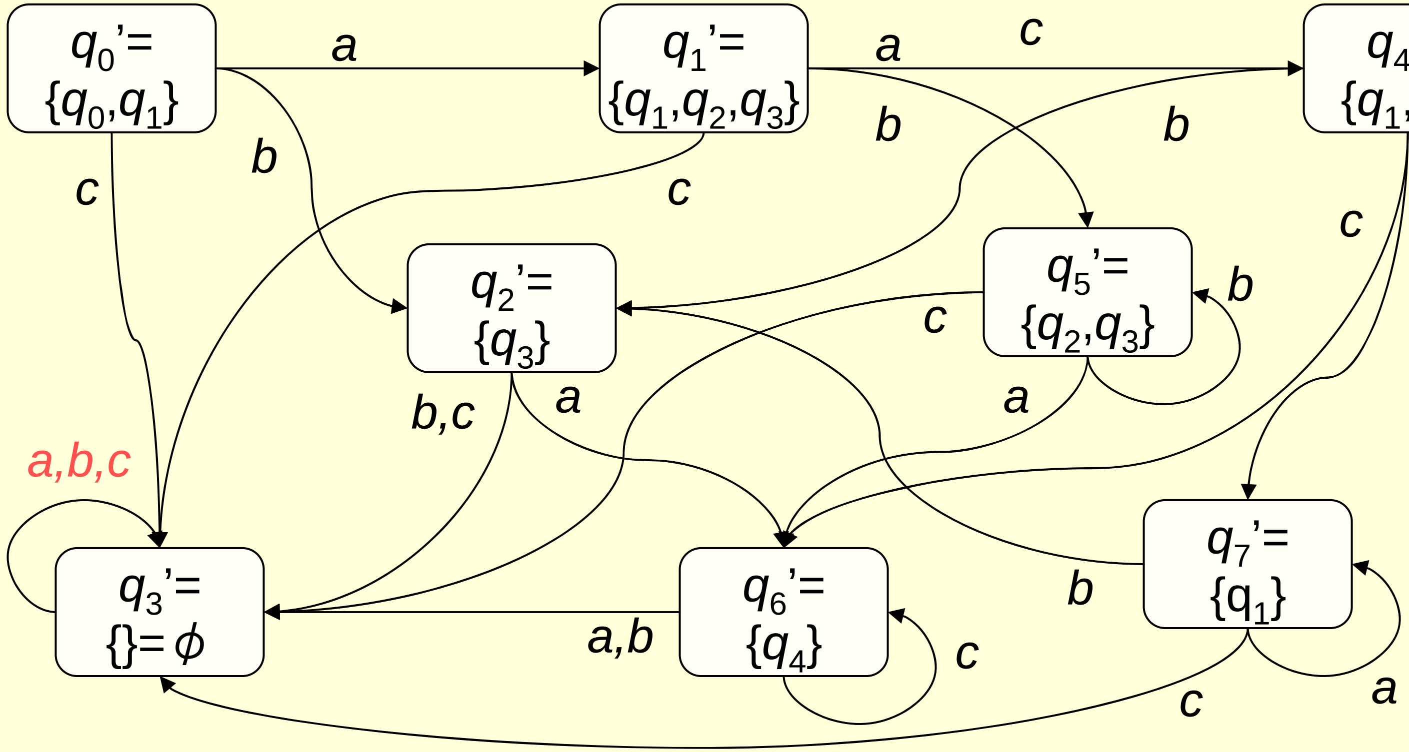
$F = \{ q_4', q_6' \}$

[結果の図示]

ε −NFA A



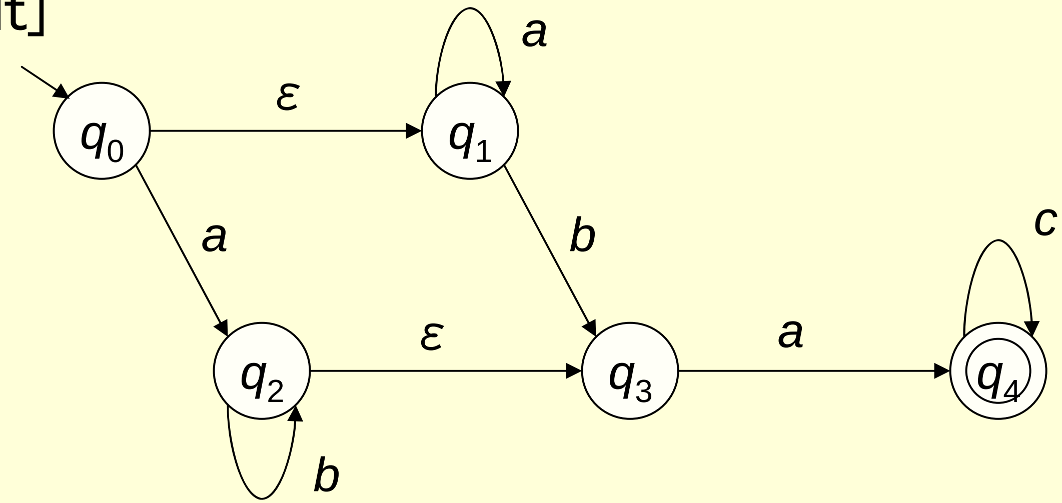
$L(A)=L(B)$を満たすDFA $B=(\{q_0', \cdots, q_7'\}, \{a,b,c\}, \delta', q_0', \{q_4', q_6'\})$



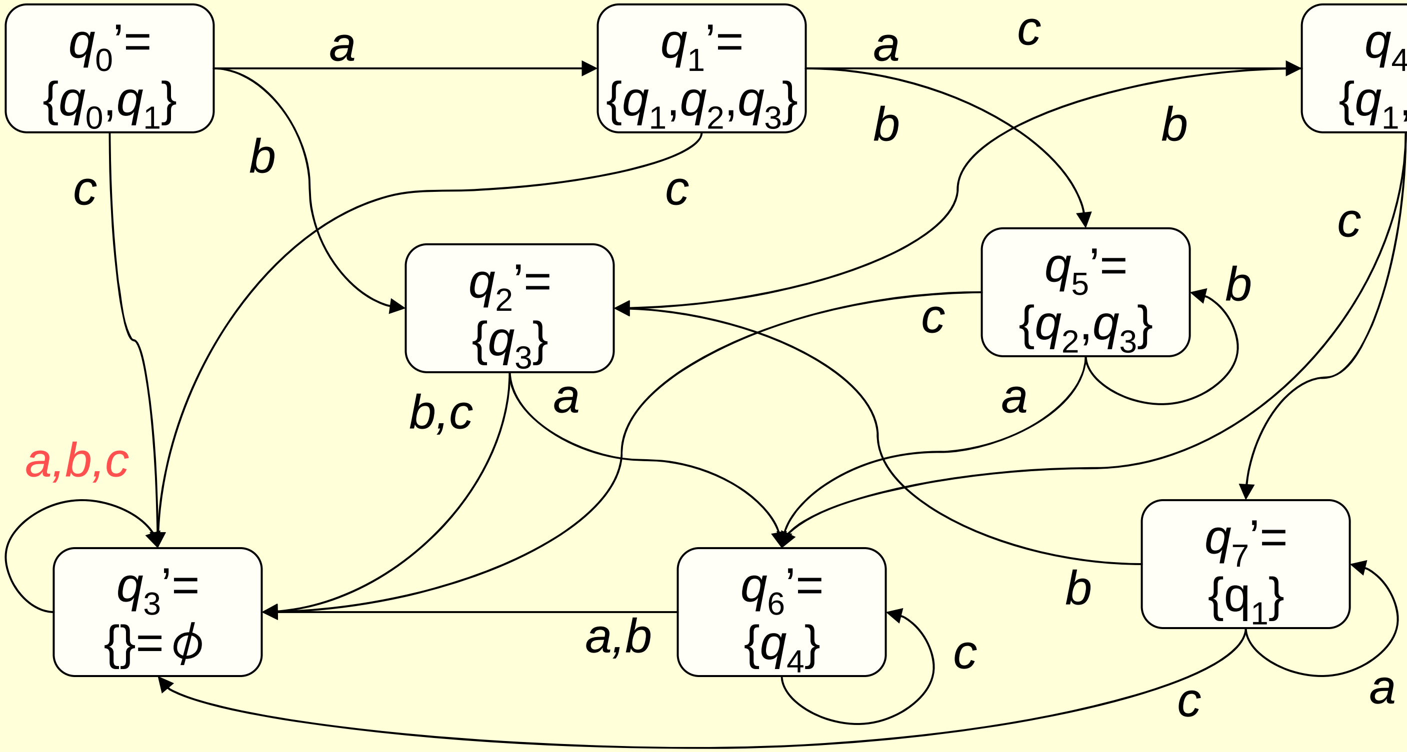
$q_0'=\{q_0, q_1\}$
$q_1' = \{ q_1, q_2, q_3 \}$
$q_2' = \{ q_3 \}$
$q_3' = \phi$
$q_4' = \{ q_1, q_4 \}$
$q_5' = \{ q_2, q_3 \}$
$q_6' = \{ q_4 \}$
$q_7' = \{ q_1 \}$

[Diagram of Result]

ε –NFA *A*



DFA $B = (\{q_0', \cdots, q_7'\}, \{a,b,c\}, \delta', q_0', \{q_4', q_6'\})$ with $L(A)=L(B)$



$q_0' = \{q_0, q_1\}$

$q_1' = \{ q_1, q_2, q_3 \}$

$q_2' = \{ q_3 \}$

$q_3' = \phi$

$q_4' = \{ q_1, q_4 \}$

$q_5' = \{ q_2, q_3 \}$

$q_6' = \{ q_4 \}$

$q_7' = \{ q_1 \}$