今日の予定

1. 展開図の基礎的な知識

1. 正多面体の共通の展開図

2. 複数の箱が折れる共通の展開図：２時間目
3. <u>Rep-Cube：最新の話題</u>

4. 正多面体に近い立体と正４面体の共通の展開図
5. ペタル型の紙で折るピラミッド型：２時間目～３時間目

Dissection of Unfolding of Cubes and Its Generalization

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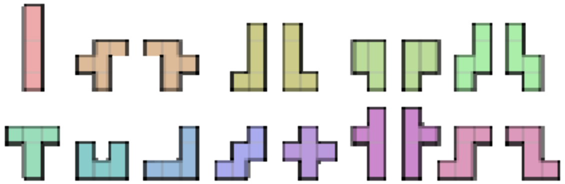
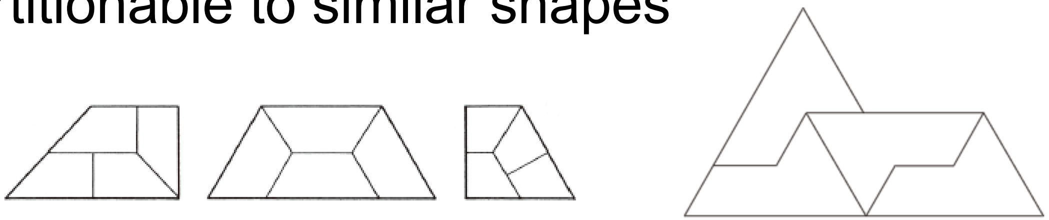
Done at 31st Bellairs Winter Workshop on Computational Geometry, Barbados, 2016

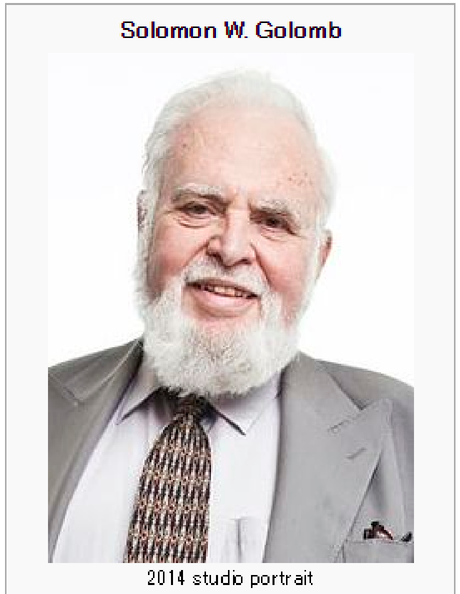
Solomon W. Golomb (1932-2016)

From the viewpoint of Recreational Mathematics, he invented Polyominoes: shapes made by unit squares



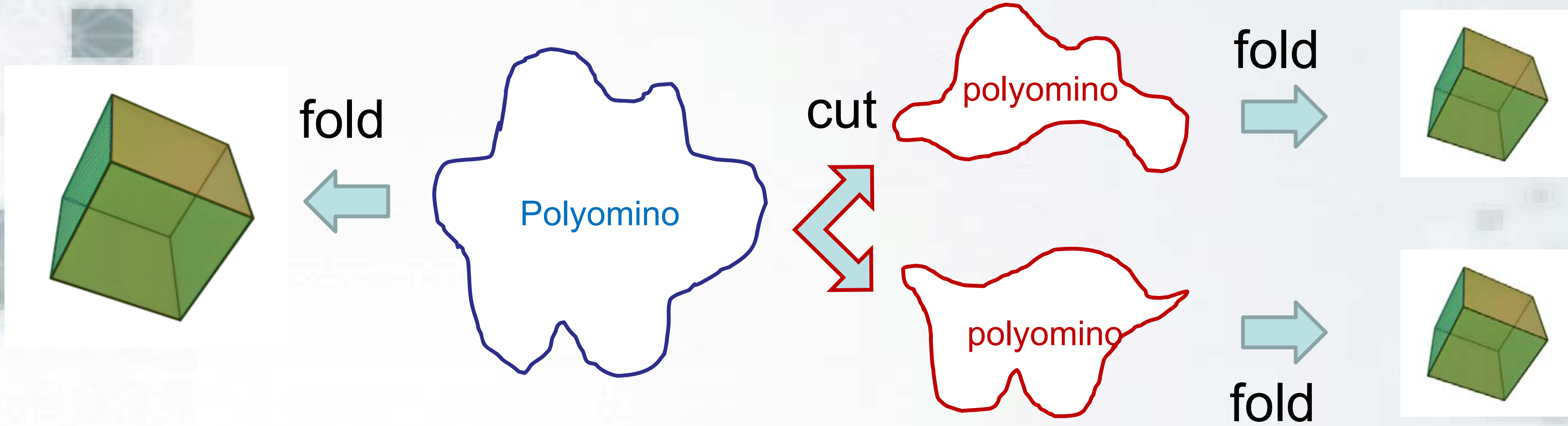
Rep-tiles: shapes partitionable to similar shapes





Extension to Folding problem

Natural Question: Is there any polyomino that folds to a cube and partitioned into some polyominoes s.t. each of which admits to fold a small cube?



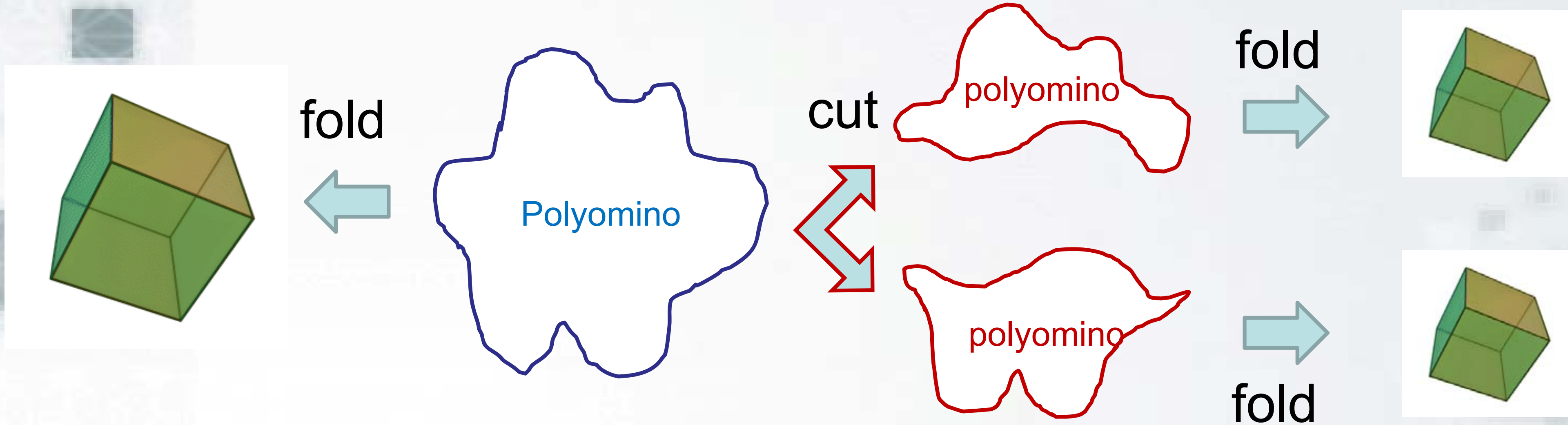
New notion: "Rep-cube"

➢ A polyomino is "rep-cube" ⇔ it folds to a cube

- of order k

  ⇔ cut into k parts s.t. each of them folds to a cube

- regular ⇔ k parts have the same size (area)



Main result

Thm 1 *There exists a regular rep-cube of order k for k = 2, 4, 5, 8, 9, 36, 50, 64.*

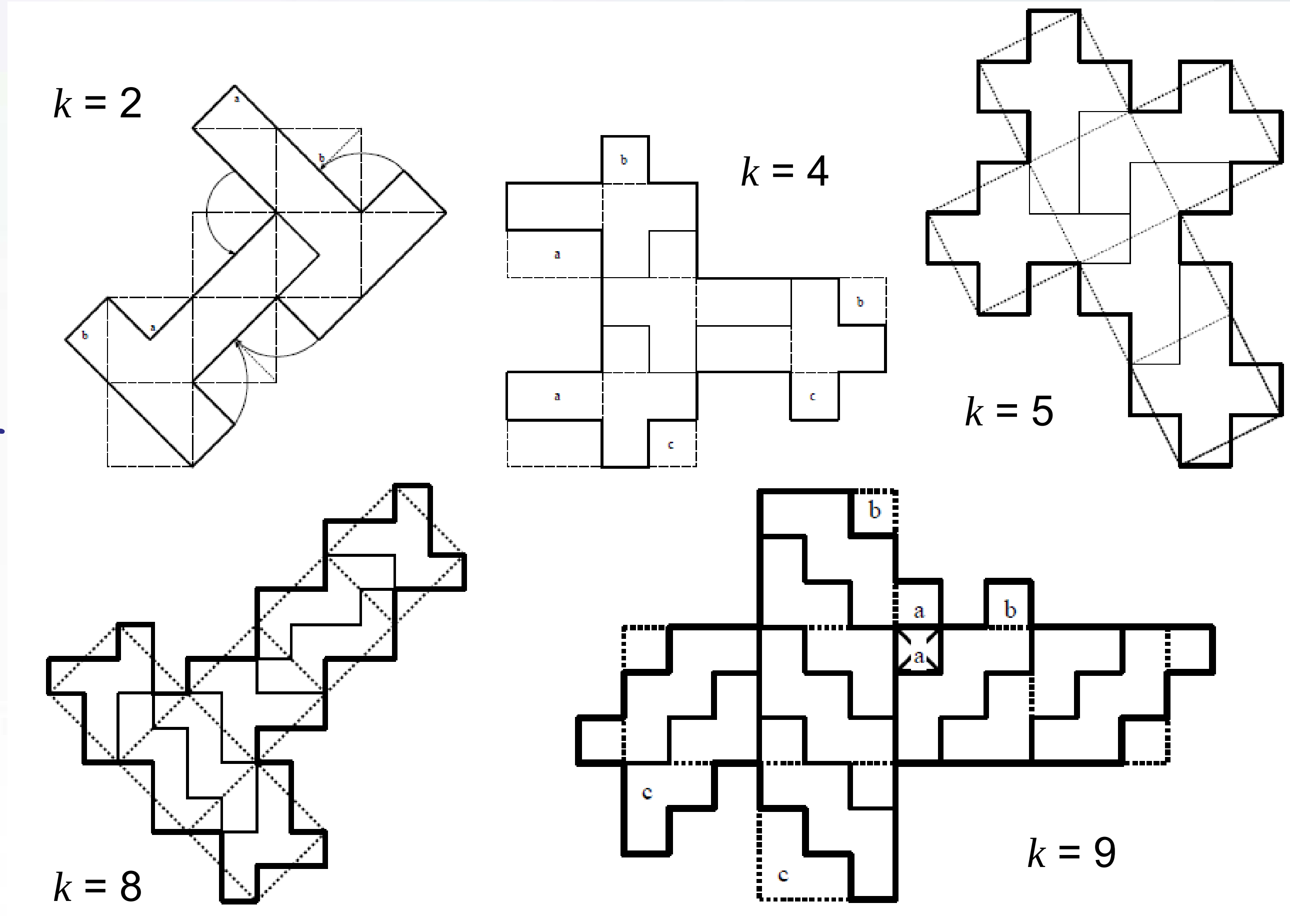
Thm 2 *There exists a regular rep-cube of order $36gk'^2$ for any positive integer k' and an integer g in* {2, 4, 5, 8, 10, 50}. *I.e., there exists an infinite number of regular rep-cubes.*

Thm 3 *There exists a non-regular rep-cube of order k for $k = 2, 10$.*

Thm 1 *There exists a regular rep-cube of order k for*

k = 2, 4, 5, 8, 9, 36, 50, 64.

Method:
Trial and Errors



Thm 1 *There exists a regular rep-cube of order k for*

k = 2, 4, 5, 8, 9, 36, 50, 64.

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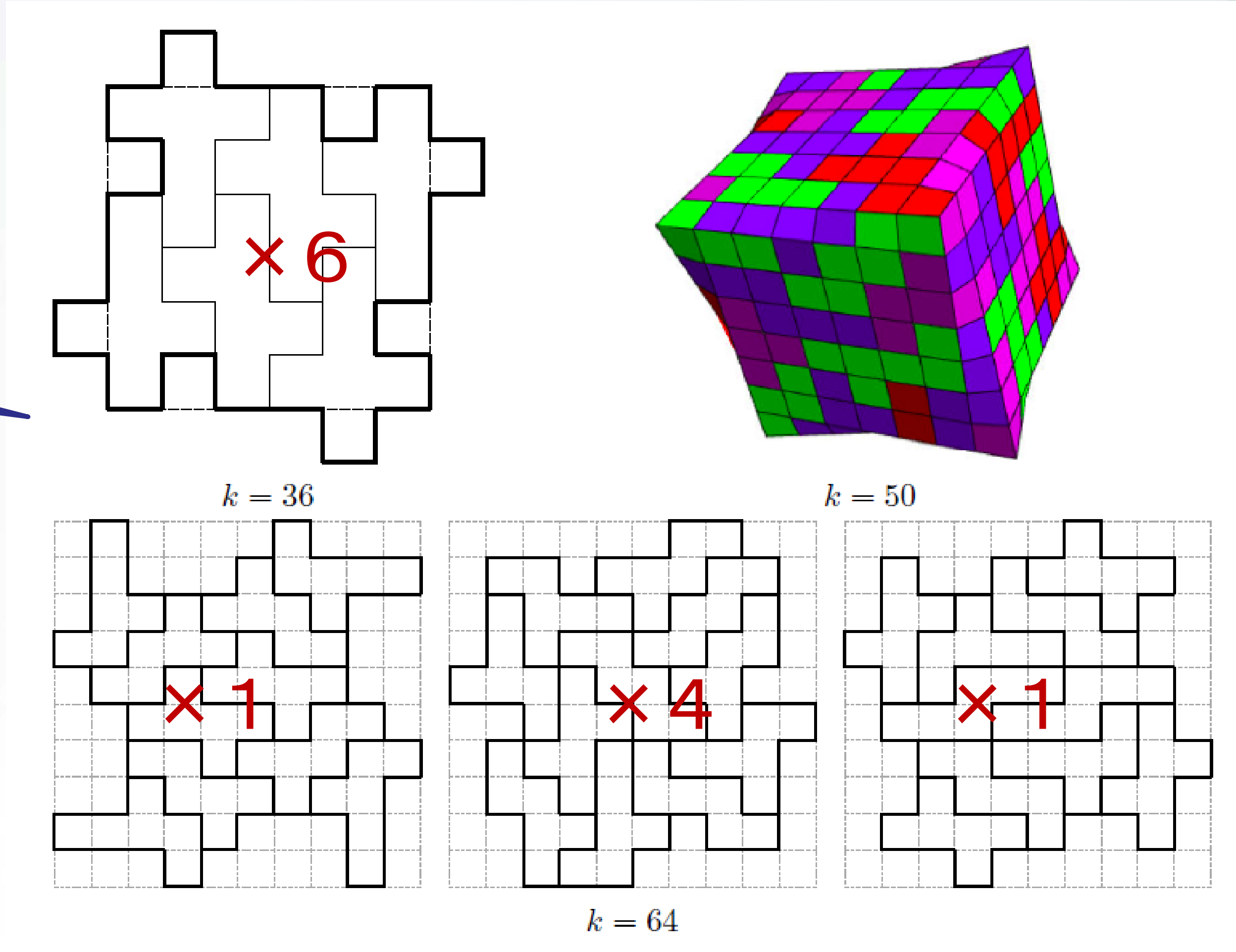
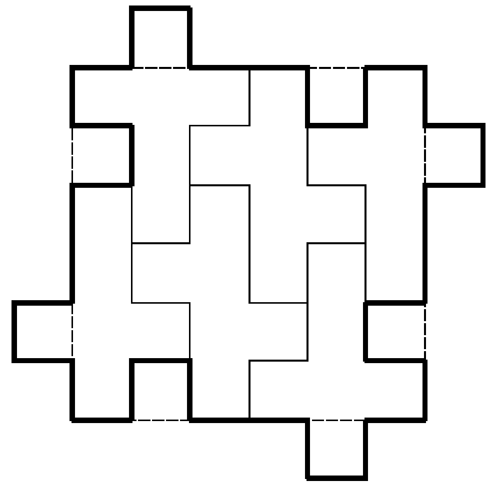


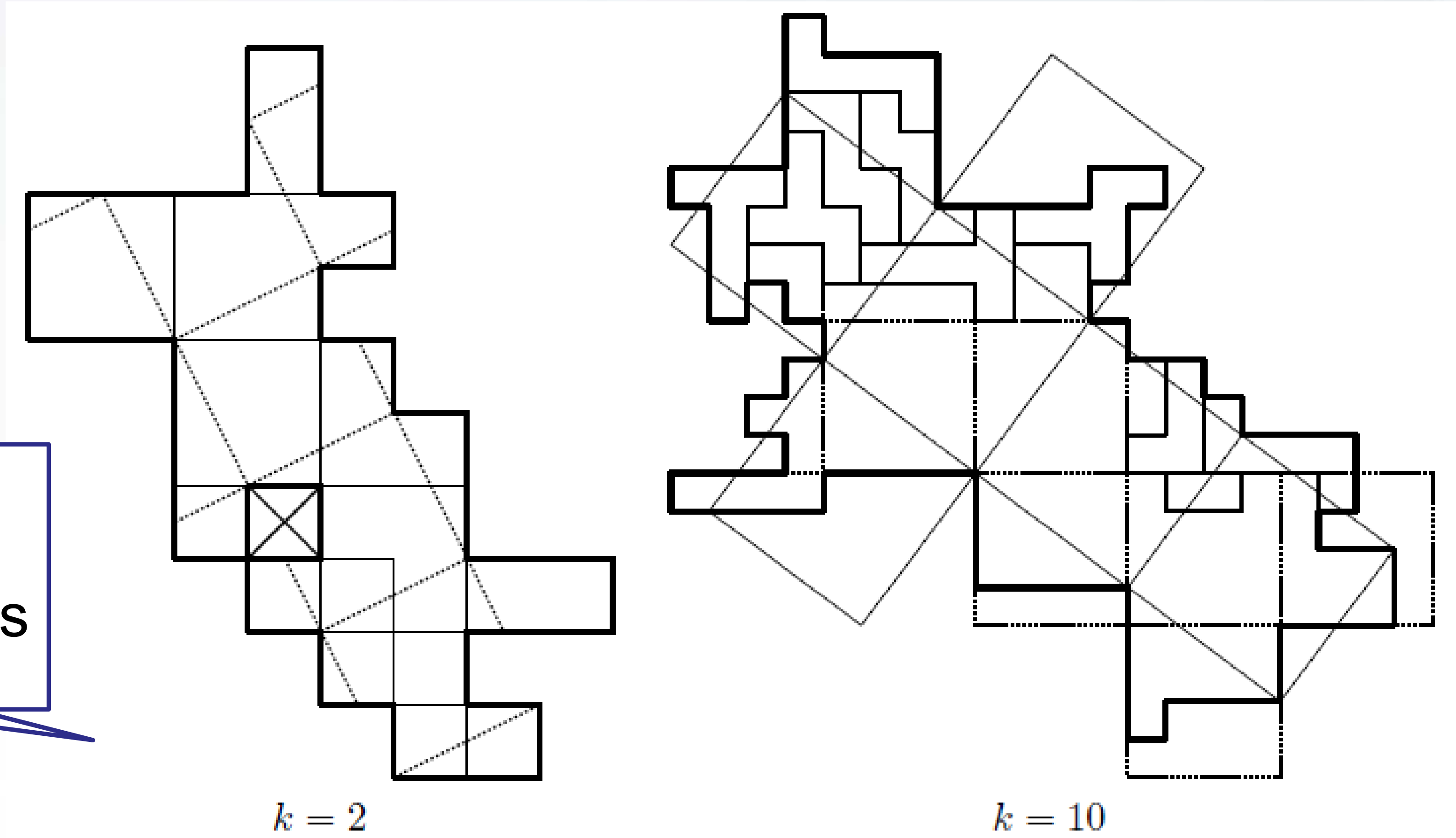
Thm 2 *There exists a regular rep-cube of order $36gk'^2$ for any positive integer k' and an integer g in* {2, 4, 5, 8, 10, 50}. *I.e., there exists an infinite number of regular rep-cubes.*

Proof Take any pattern in Thm 1. Then replace each unit square by the right pattern for k=36 in Thm 1. We can repeat it recursively any times.



$k = 36$

Thm 3 *There exists a non-regular rep-cube of order k for $k = 2,\ 10$.*



Method:
Trial and Errors

Future work

Thm 1 *There exists a regular rep-cube of order k for*

k = 2, 4, 5, 8, 9, 36, 50, 64.

Thm ... *for*

So far, these patterns in Theorems 1 and 3 are given by just trial and errors!!
We need something more…

Thm 3 *There exists a non-regular rep-cube of order k*

for $k = 2, 10$.

Generalization to 2D

Basic Idea;

C.f. Not polyominoes any more…

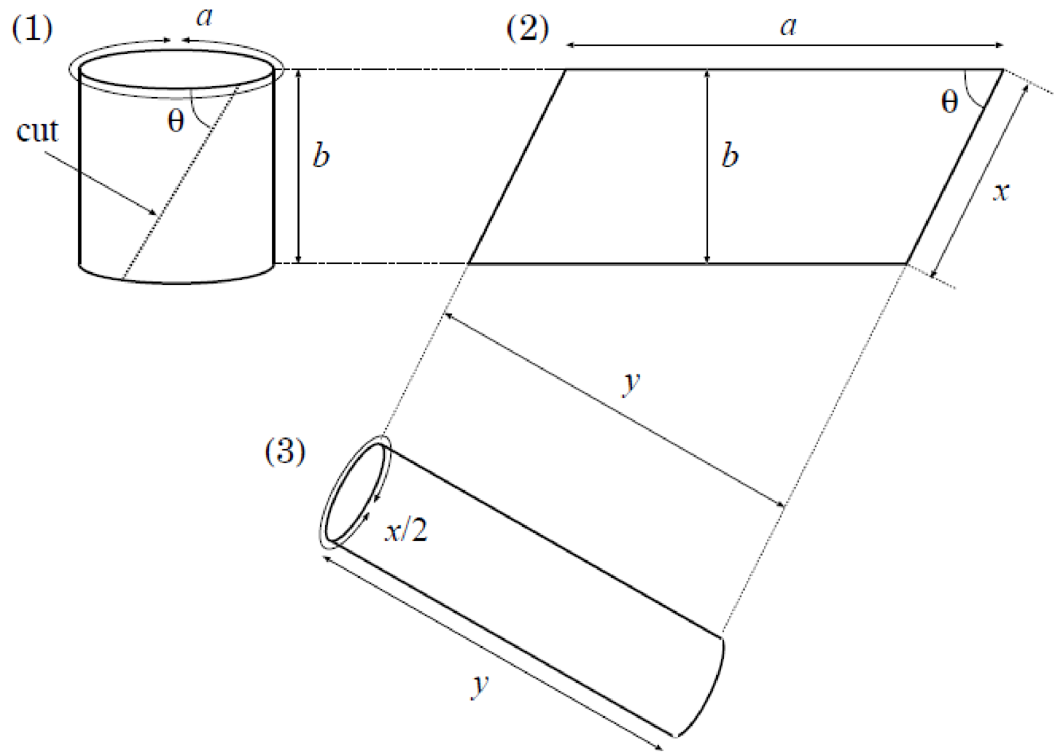
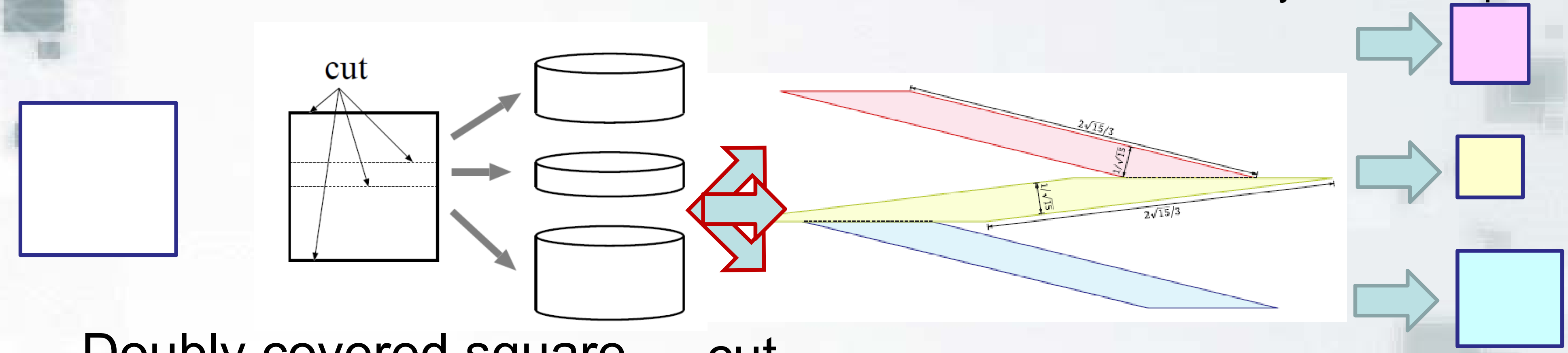


Fig. 6 (1) A cylinder of circumference a and height b, (2) a common development of two cylinders, (3) the other cylinder of circumference $x/2$ and height y.

Generalization to 2D

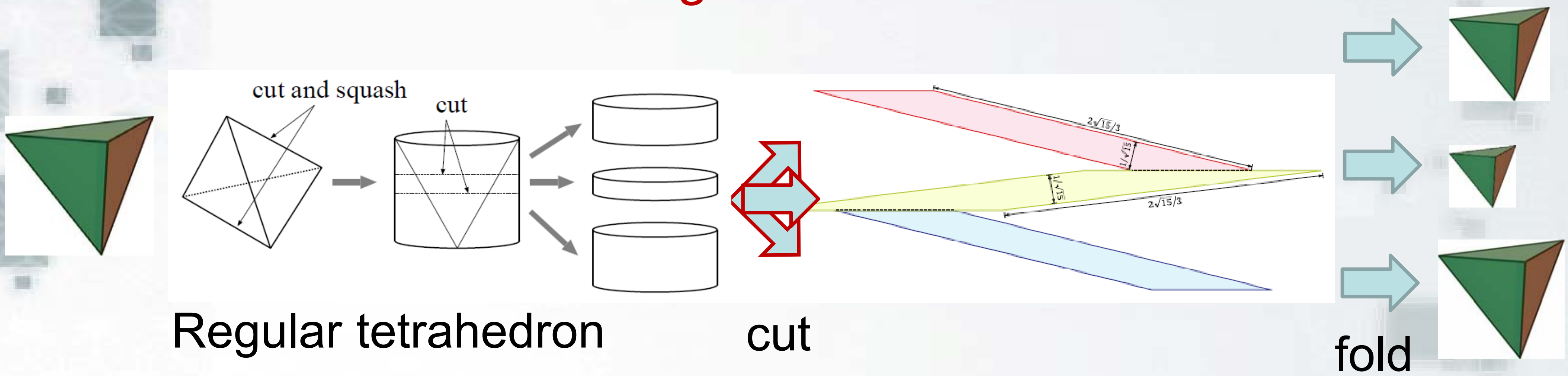
Thm 4 *For any positive real numbers A, $a_1, a_2, \ldots, a_k$ such that $\Sigma_i\, a_i = A$, there is a net of a doubly-covered square with area A that can be cut into k polygons with areas $a_1, a_2, \ldots, a_k$, each of which can be folded into a doubly-covered square.*



Return to 3D

Thm 5 *For any positive real numbers $A, a_1, a_2, \ldots, a_k$ such that $\Sigma_i\, a_i = A$, there is a net of a regular tetrahedron with area A that can be cut into k polygons with areas $a_1, a_2, \ldots, a_k$, each of which can be folded into a regular tetrahedron.*

Regular tetrahedron

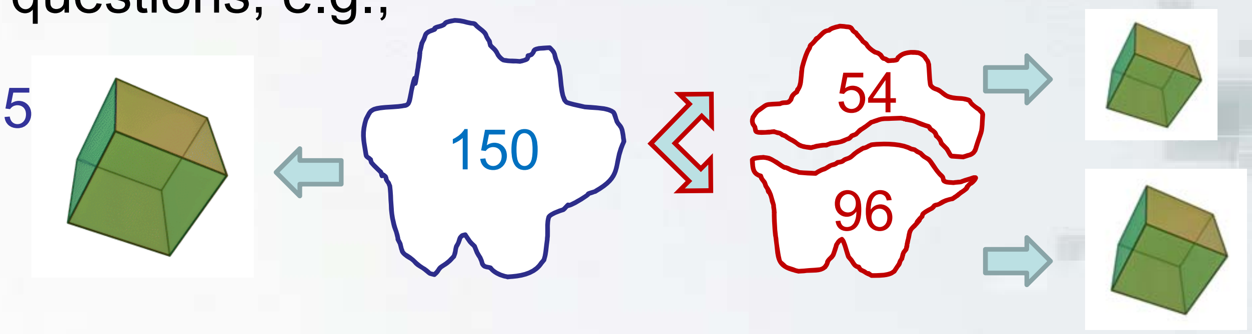


Conclusion and Future work

Conclusion: We introduce a new notion of rep-cube

- We have many *examples*
- Theoretically, there exist infinitely many
- We can consider many variants/generalizations
  - Many open questions; e.g.,

General method for any Pythagorean triple…?



Future Work

- We need more theoretical work/results?
- Applications … not only recreational math?