A survey on computational complexity of finding good folded state with few crease width

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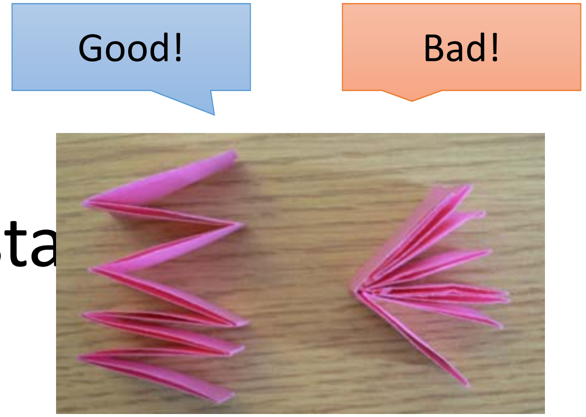
- Erik D. Demaine, David Eppstein, Adam Hesterberg, Hiro Ito, Anna Lubiw, Ryuhei Uehara and Yushi Uno: Folding a Paper Strip to Minimize Thickness, *The 9th Workshop on Algorithms and Computation (WALCOM 2015)*, Lecture Notes in Computer Science Vol. 8973, pp. 113-124, 2015/02/26-2015/02/28, Dhaka, Bangladesh.
- Takuya Umesato, Toshiki Saitoh, Ryuhei Uehara, Hiro Ito, and Yoshio Okamoto: Complexity of the stamp folding problem, Theoretical Computer Science, Vol. 497, pp. 13-19, 2012.
- Stamp foldings with a given mountain-valley assignment in *ORIGAMI*[5], Ryuhei Uehara, pp. 585-597, CRC Press, 2011.

- Good Problem = 1D Origami

1. Stamp Folding: How many folded sta
    1. 1 dimensional
    2. Unit length between creases
    3. Any MV pattern
2. Folding Complexity: How many pattern?
    1. 1 dimensional
    2. Unit length between creases
    3. Given MV pattern
3. Crease width: How thin can we fold for given MV pattern?
    1. 1 dimensional
    2. (Unit length)/(general length) between creases
    3. Given MV pattern



__Crease width__ at a crease
↕
# of paper layers at a crease

Why we investigate these problems in 1D?

- From the viewpoint of Computer Science

Goal: Estimate the complexity of an origami design

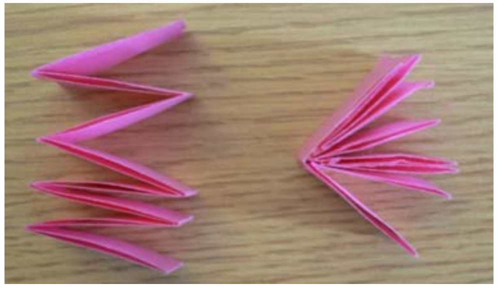
  1. Time: the number of "steps" of computation
  2. Space: the number of memory cells required to compute
     - We have time-space tradeoff;
       Complexity of a problem = time × space
- In Origami Science
  - Two resources of an origami model;
    1. Time: the number of "folding operations"
    2. Space: ?? Crease width
       - We have a kind of time-* tradeoff;
         Complexity of an origami design= time × crease width ～ accuracy

To fold fast, we pile many paper layers, which causes large crease width, that means in accuracy!!

Minimization of Crease width

Good!

Bad!



Input: Paper of length $n+1$ and $s \in \{M, V\}^n$

Output: folded paper according to s

Goal: Find a *good* folded state with *few* *crease width*

- At each crease, the number of paper layers between the paper segments hinged at the crease is *crease width* at the crease
- Two minimization problems;
  - minimize maximum
  - minimize total (=average)

No!!

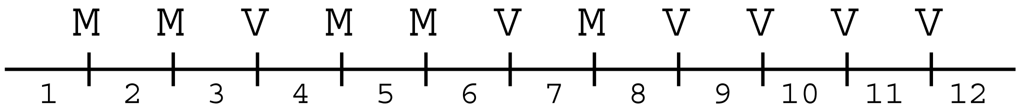
It seems simple, … so easy??

Crease width problem

Cf. I’d checked that by brute force…

Simple non-trivial example (1)

Input: MMVMMVMVVVV

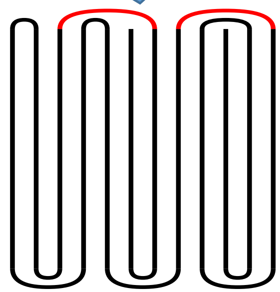


The number of feasible folded states : **100**

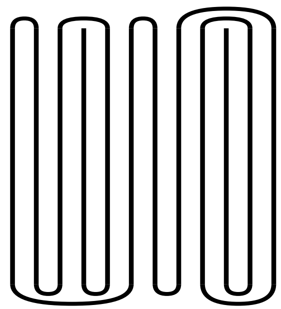
Goal: Find a *good* folded state with few *crease width*

- The **unique** solution having min. max. value 3
  [5|4|3|6|7|1|2|8|10|12|11|9]

total=13



- The **unique** solution having min. total value 11
  [5|4|3|1|2|6|7|8|10|12|11|9]



Crease width problem

Simple non-trivial example (2)

A few facts;

- a pattern has a <u>unique folded state</u> iff it is <u>pleats</u>
- solutions of {min max} and {min total} are different depending on a crease pattern.
- there is a pattern having *exponential* combinations

This pattern is almost pleats, but it has exponentially many folded states....

Known results (USUIO2012)

The crease width problem (unit interval)

- Min-Max problem: NP-complete

  (Reduction from 3-Partition)

- Min-Total problem:
  - Complexity is *still open*...
  - *Fixed Parameter Tractable algorithm*;

    it runs in $O((k+1)^k n)$ time, where k is the total crease width.

    the algorithm itself is natural, but analysis is bit tricky.

It is intractable even for small n...

It is solvable if k is quite small...

MinMax is NP-complete

Proof: Polynomial time reduction from 3-Partition.

<u>3-Partition</u>:

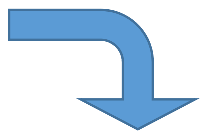
$(B/4 < a_j < B/2)$

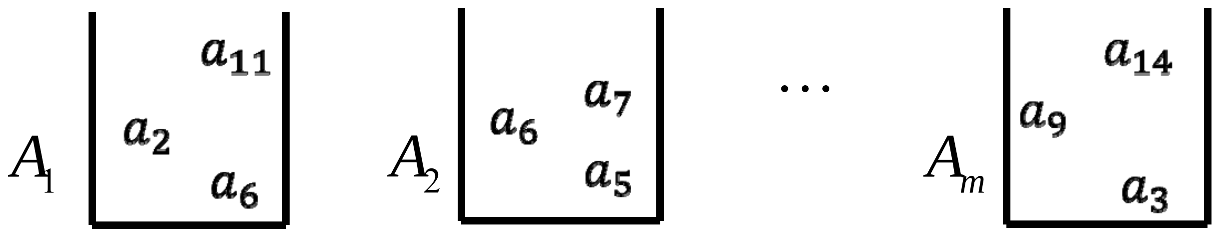
Input: Set of integers $A = \{a_1, a_2, \ldots, a_{3m}\}$ and integer B

Question: Is there a partition of A to $A_1, \ldots, A_m$

such that $|A_i| = 3$ and $\sum_{a_j \in A_i} a_j = B$

$A = \{a_1, a_2, \ldots, a_{3m}\}$

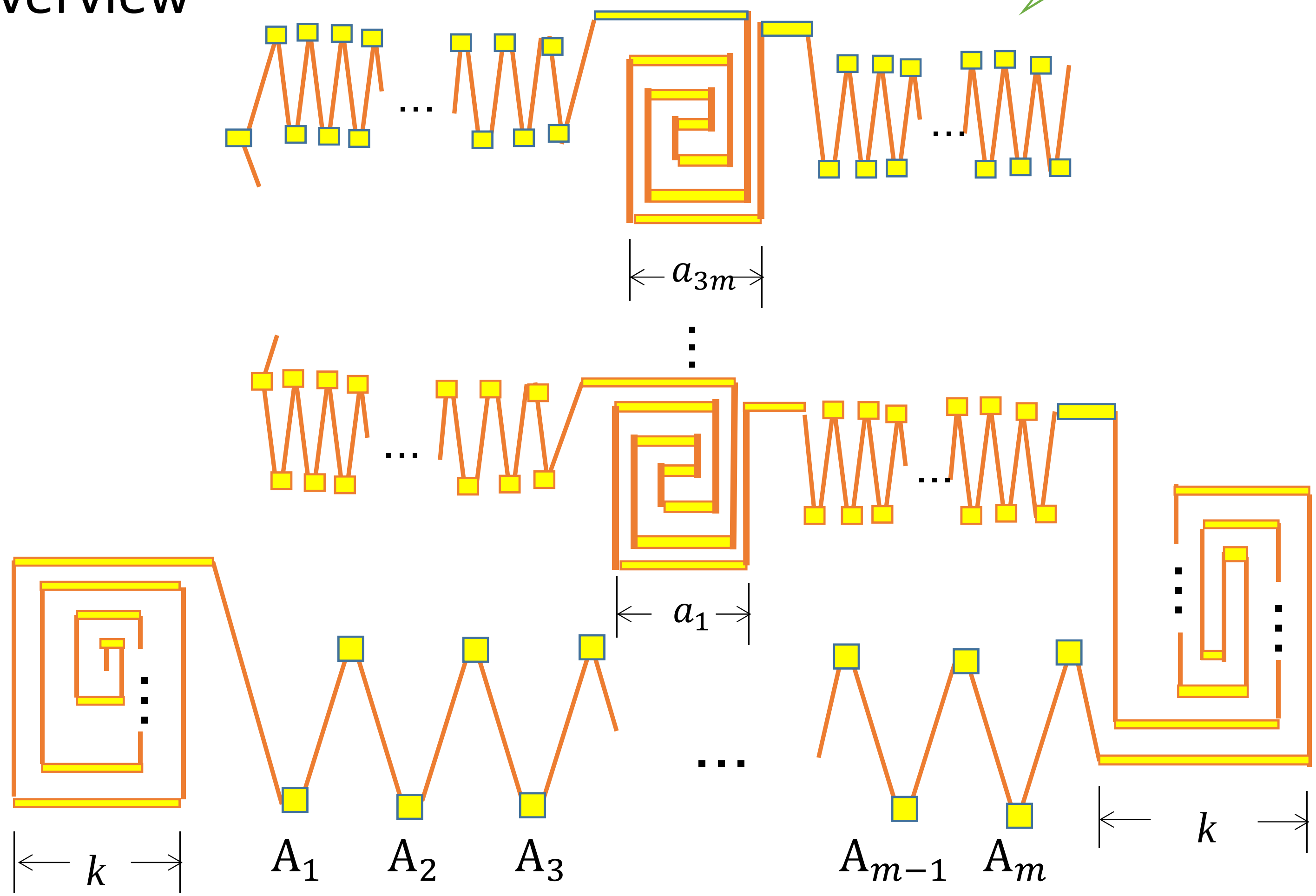




Construction for MinMax

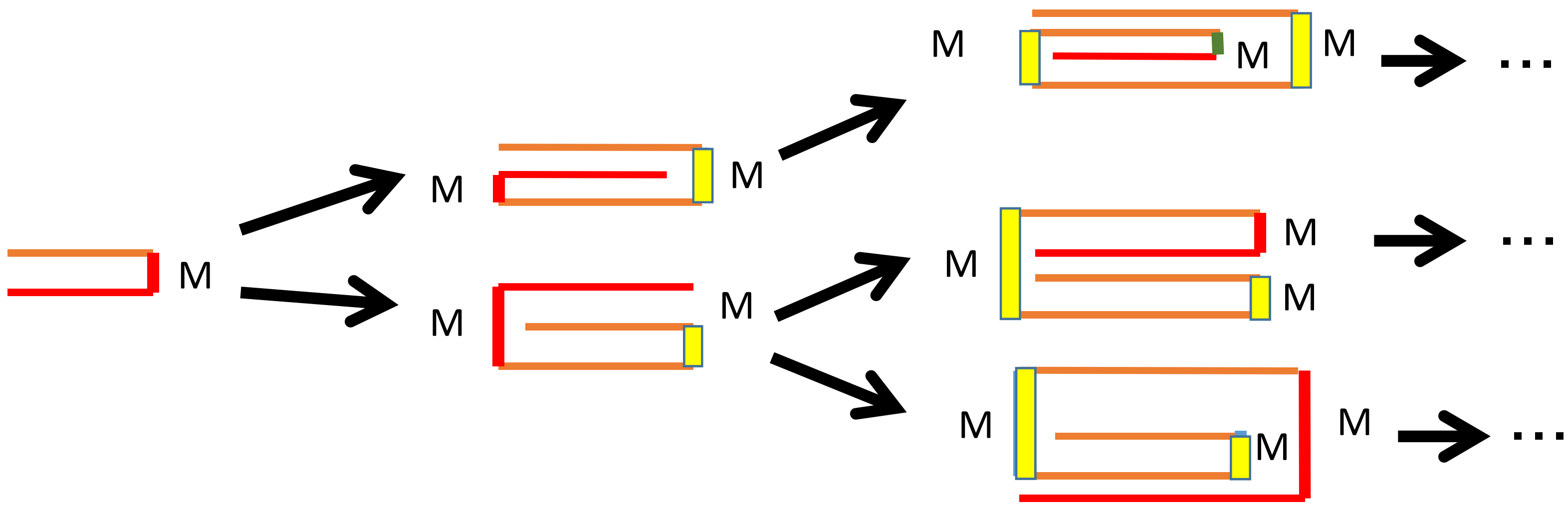
Overview

図がわかりにくいのであとで再放送。。。



(Poly-time) Algorithm for MinTotal

- Enumerate all folding ways with respect to the string s up to total crease width k.
- Each folded state is generated incrementally.



- Check the total crease width at each increment.

Running time

- The algorithm for given fixed total crease width k runs in $O(n^{2+k})$ time.
  - at each crease, the sequence of c.w. is
    - $c_1, c_2, \ldots, c_i$, … with

$$\sum_{i=1,2,\ldots} c_i \leq k$$

  - that is a partition of ≦ k
- With more careful (and complex) analysis shows that the algorithm runs in $O((k+1)^k\, n)$ time!!

That is, this is fixed parameter tractable algorithm!

Known results (USUIO2012)

We chosen this at Barbados in 2014

Possible extensions in 2012;

1. Non-unit intervals between creases

   How can you measure the thickness of pile of various lengths?

2. 2-Dimenaional (…related to Map-folding?)

   How can you measure the crease-width in 2D?

3. Different Criteria for "space complexity"

   We have few ideas…

   (by Prof. Iwama: you can fold left $<k$ and right$>k$ creases in $[1..n]$)

   (Area to fold for long-pipe folding)

Now we turn to・・・ (DEHILUU 2015)

- Quite simple model
  1. 1 dimensional
  2. ~~Unit length between~~ creases

Non-unit intervals!!

Not only M/V, but also lengths between creases are given

But,,, wait,,,

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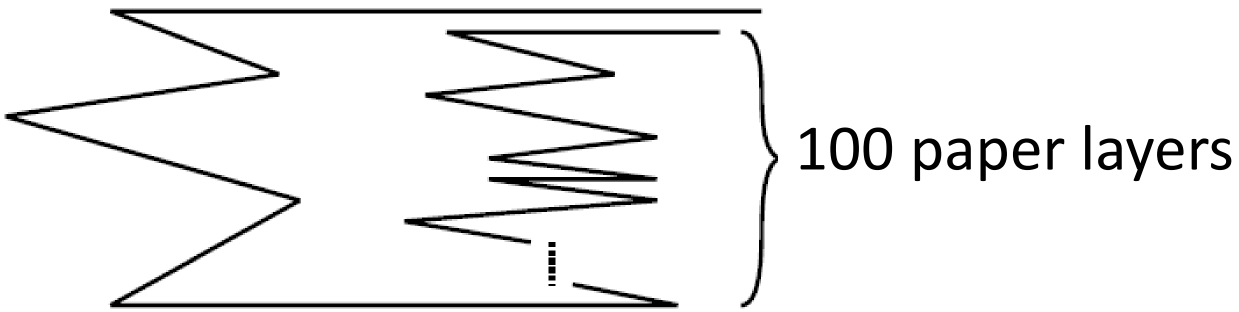
For non-unit interval creases...

Crease width = the number of paper layers at a crease?



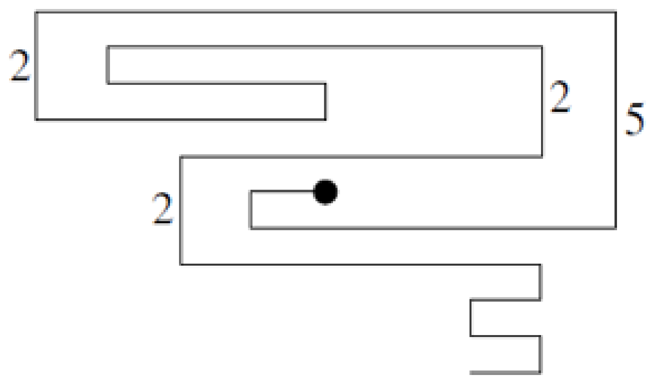
How can we count the paper layers?

For non-unit interval creases…

Crease width = the number of paper layers at a crease?

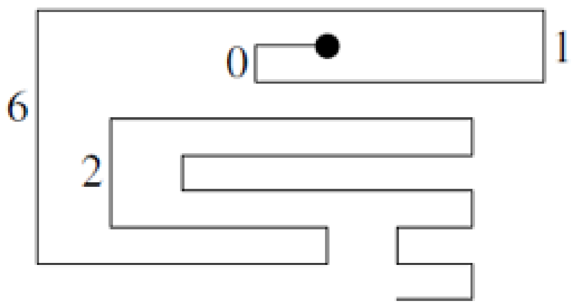
- We introduce three new "widths" of a folded state:
  - Two are natural extensions of Max-CW and Total-CW; one is totally new!
- For VMVMVVMMMM, e.g., we have;

Minimum
max crease width



$h = 11$, $m = 5$, $t = 11$

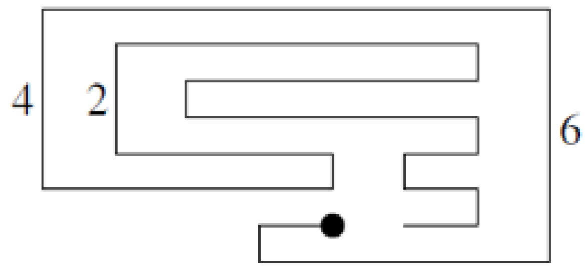
Minimum
total crease width



$h = 9$, $m = 6$, $t = 9$

New

Minimum **height**



$h = 8$, $m = 6$, $t = 12$

Main results in (DEHILUU 2015)

Summary

	Unit interval model in [USUIO2012]	General model in (DEHILUU 2015)
max crease width	NP-complete	⇒ NP-complete
total crease width	open	NP-complete [DEHILUU 2015]
height	trivial	NP-complete [DEHILUU 2015]

Proof Idea

Minimize height is NP-complete

Proof: Polynomial time reduction from 3-Partition.

<u>3-Partition</u>:

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Input: Set of integers $A = \{a_1, a_2, \ldots, a_{3m}\}$ and integer B

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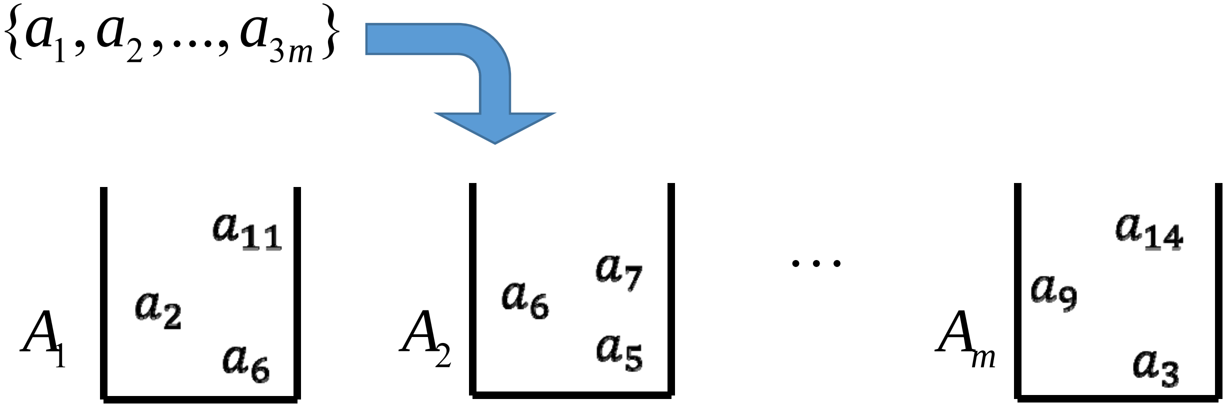
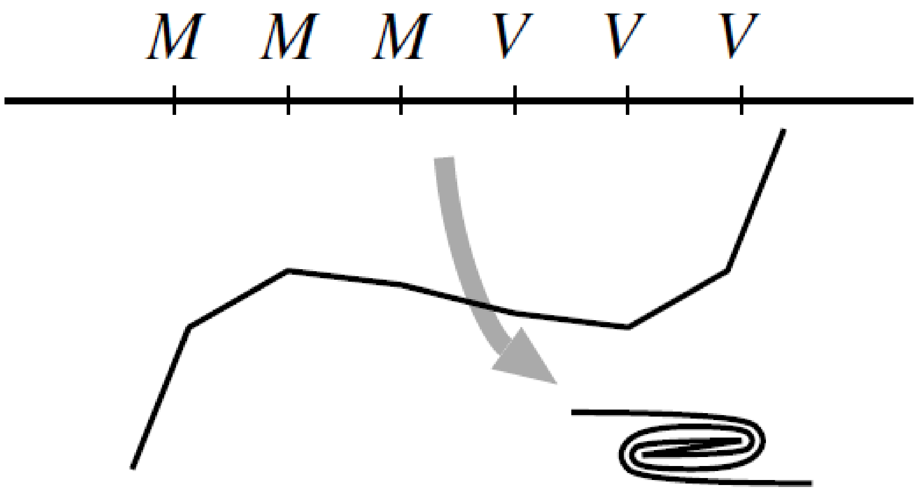


Minimize height is NP-complete

Proof: Polynomial time reduction from 3-Partition.

<u>Basic gadget</u>

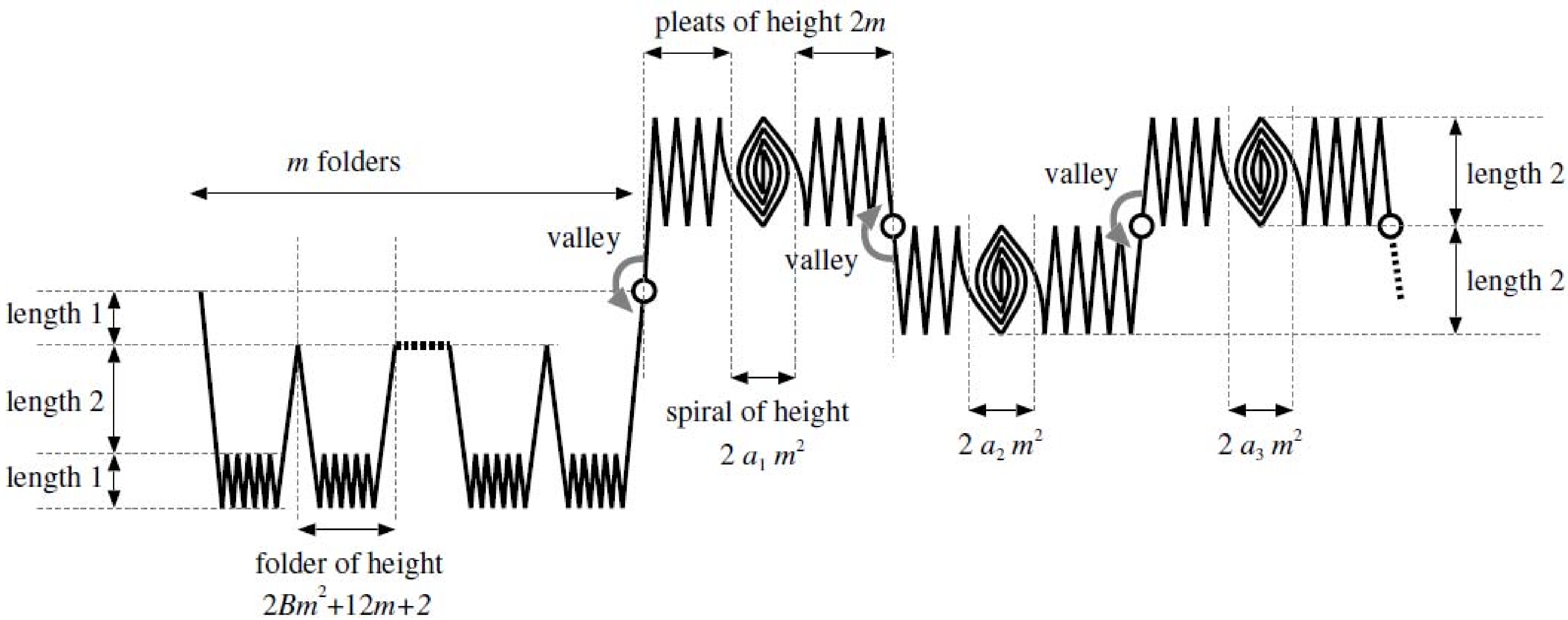


The way of folding is unique by bit longer end-edges

Minimize height is NP-complete

Proof: Polynomial time reduction from 3-Partition.

Overview



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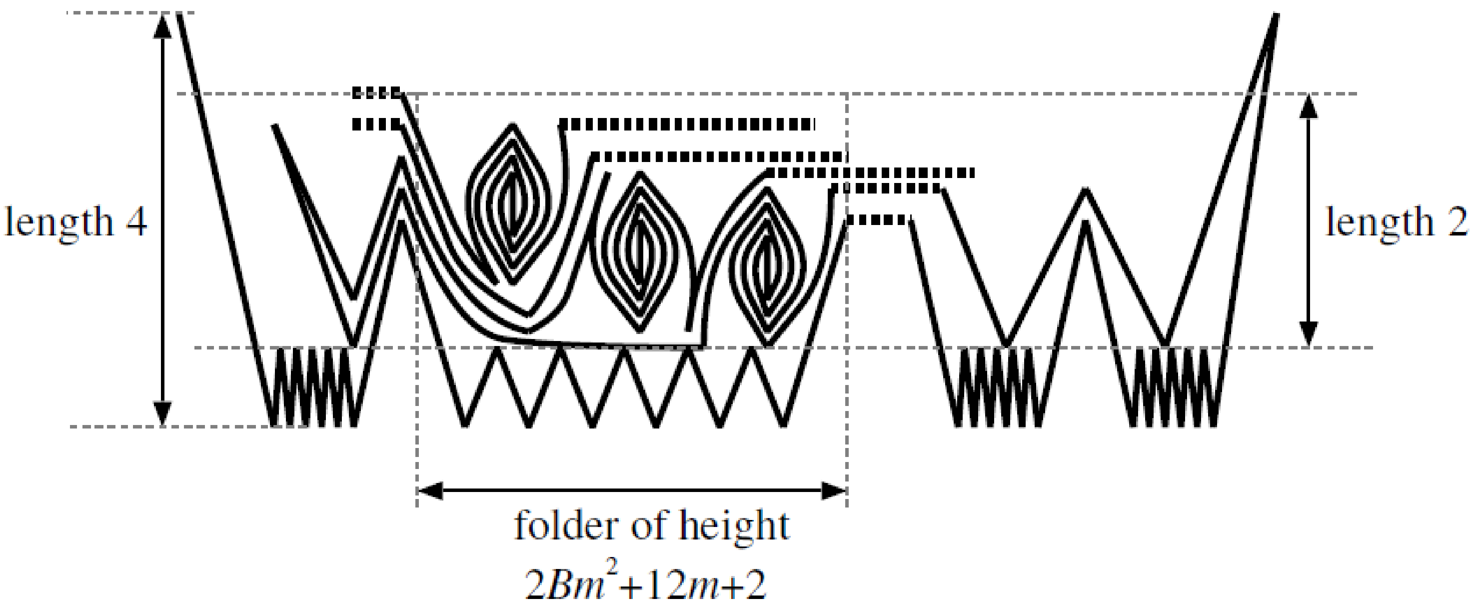

Summary & Future work…

Origami is interesting even in 1 dimension!!

	Unit interval model in [USUIO2012]	General model in (DEHILUU 2015)
max crease width	NP-complete	⇒ NP-complete
total crease width	open	NP-complete [DEHILUU 2015]
height	trivial	NP-complete [DEHILUU 2015]

Future work:

- Replace "open" into ???
- Extension to 2 dimension
  - Different measures of "thickness"?
- Estimation of the way of folding (~time complexity)
- Nicer model for "*Time-space trade off*" for Origami