Computational Origami

Ryuhei Uehara
Japan Advanced Institute of Science and Technology (JAIST)
School of Information Science
uehara@jaist.ac.jp
http://www.jaist.ac.jp/~uehara

I628E Information Processing Theory

- Schedule
  - January 27 (13:30-15:10)
    - Introduction to Computational Origami
    - Polygons and Polyhedra folded from them
  - January 29 (10:50-12:30)
    - Computational Complexity of Origami algorithms
  - February 3 (9:00-10:40)
    - Advanced topics
    - 13:30-15:10 (Office Hour at I67-b)

I628E Information Processing Theory

- Report (up to 20pts)
  - Submit a report about one of the following two options:
  1. Survey some paper(s) appearing in these three lectures
  2. Solve some problems appearing in these three lectures

  - Firm deadline: 17:00, February 10 in one of the following two ways
    - By email:
      PDF file (word file is not acceptable) from JAIST account.
    - By paper:
      A4 size paper, staple at the top-left corner.

    You can write your report in English or Japanese.

Origami?

- In Japanese, "Ori"=folding and "kami/gami"=paper.
  - It was born in 1500? with inventing paper, in some Asia, maybe. Of course, we have no record on paper!
  - Now, "ORIGAMI" is an English word, and there are some shelves in bookstores in North America and Europe.
  - Origami-like things…

There are some "Origami"s which are not folded, and not paper any more now a day!! Maybe by an NSF big fund?

Origami as paper folding

- Normal Origami
- Difficult Origami
- Impossible Origami (for most human!)

Kawasaki Rose

Maekawa Devil

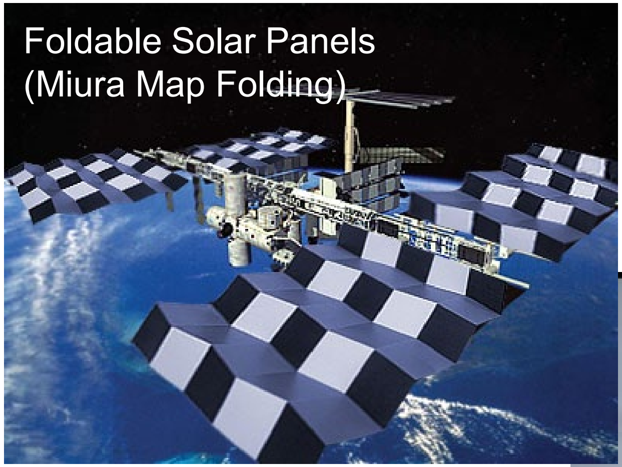
By Tetsushi Kamiya (Origami Champion)

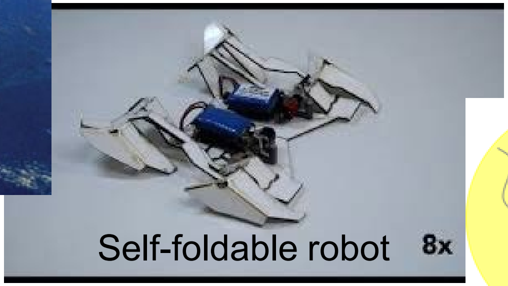
Applications of Origami

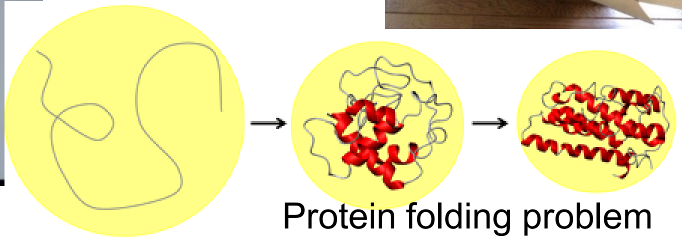
- There are many applications of "Folding"→ Computational Origami

Science based on the basic operations of "folding"

There are many applications and open problems of "folding"







Computational ORIGAMI?

- Rapid development of Origami itself
  - “Complex Origami” were born in 1980s-1990s

Maekawa Devil (1980)
Folded from a square paper!

Kawasaki Rose (1985)
Folded from a square paper!

Cuckoo clock by Robert Lang (1987)
Folded from a rectangular Paper of size 1 × 10!

Computational ORIGAMI?

- Computer Aided Design of Origami
  - Development of origami design by computer since 1990s

Cuckoo clock by Robert Lang (1987)
Folded from a rectangular Paper of size 1 × 10!

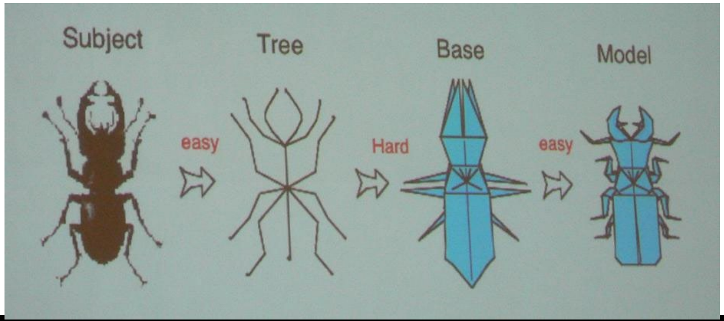
Origamizer by Tomohiro Tachi (2007)
Folded from a rectangular paper in 10 hours!

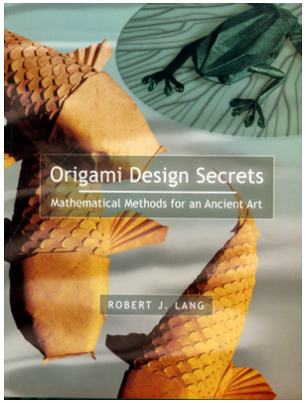
Rotational symmetry origami by Jun Mitani (2010)
Folded from a rectangular paper

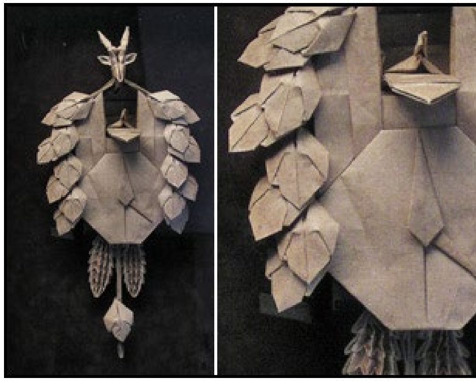
Research on origami in Computer Science

- Popular rather than US/Europe, not Asia ;-)
- Development of software with methodology:
  - 1980s: Maekawa Devil
    - Origin of "complex origami"
    - Designed by CAD-like method (by his hand)
  - 2000s: TreeMaker by R. Lang
    - It develops a given metric tree onto a square
    - It solves some <u>optimization problems</u> in a practical time

…including NP-Complete problem







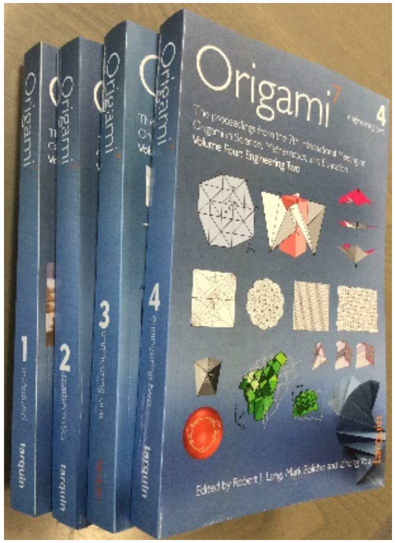
International Conference on Origami Science

1. 1989@Italy
   - International meeting of Origami Science and Technology
2. 1994@Japan
   - International meeting of Origami Science and Art
3. 2001@USA
   - 3OSME(International meeting of Origami Science, Mathematics, and Education)
4. 2006@USA
   - 4OSME
5. 2010@Singapore
   - 5OSME
6. 2014@Japan
   - 6OSME
7. 2018@UK
   - 7OSME

Proceedings is on market

Proceedings become 2 volumes

Proceedings become 4 volumes



Computational ORIGAMI?

- Proposal of “Computational Origami”

Since 1990s:

In Computational Geometry area, they consider “folding problems” as problems in computational geometry and/or optimization

The BIG name in this area: Prof. Erik D. Demaine

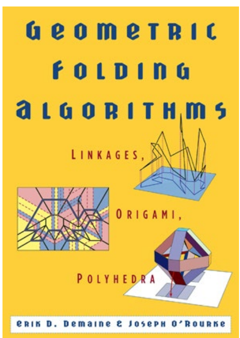
- Born in 1981
- Got Ph.D in Canada when he was 20 years old, and a faculty position at MIT.
- Topic of his thesis was Computational Origami!

Computational ORIGAMI

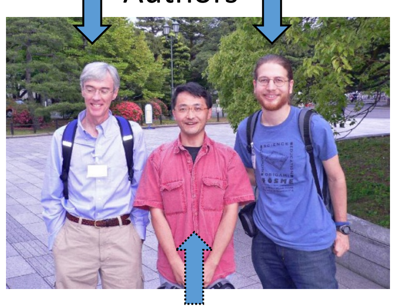
- Bible:

  Geometric Folding Algorithms: Linkages, Origami, Polyhedra

  by J. O'Rourke and E. D. Demaine, 2007.

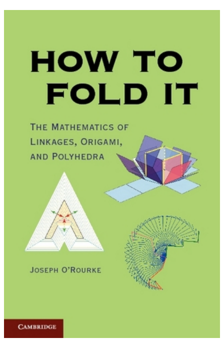


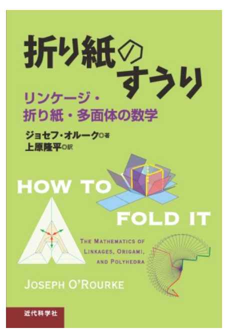
Authors



I, translated (2009).

2018





Most results can be found in this book…

2011

2012

Topics in the lectures

Computational Geometry

Part 1: Polygons and polyhedra folded from them

- Relationship between unfolding and solids: Big open problem
- How can we compute (convex) “polyhedra” from a given “unfolding”?
  - Mathematical characterization/algorithms/computation power

Algorithms and Computational Complexity

Part 2: Algorithms and computational complexity of “folding”

- Basic operations of origami
- Algorithms and complexity of origami
  - Efficiency of folding of 1-dimensional origami (algorithms and complexity)
    - Efficient algorithm (how can we reduce the number of folding?)
    - How can we evaluate “good” folded states?

Part 3: Recent topics

There are many open problems, where many young researchers working on

Today…

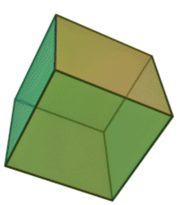
1. Basic facts for unfolding
2. Polygons foldable two or more boxes
3. Common unfolding of regular polyhedra (or Platonic solids)

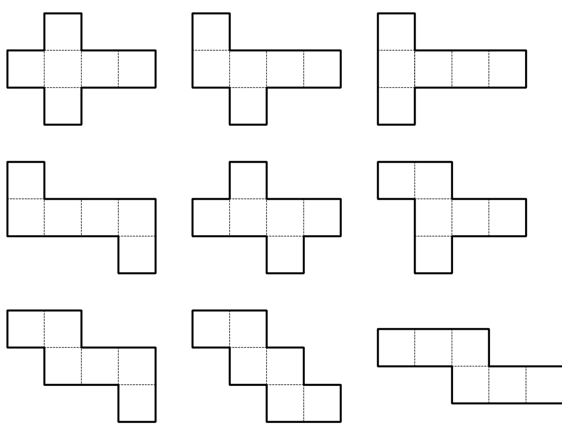
Preparation: Unfolding?

- (General) Unfolding: Cut the surface of a polyhedron along line segments and unfold it onto a plane
  - It should be connected
  - It should be a simple polygon without self-overlapping
- Edge-Unfolding: Unfolding obtained by cutting along only edges

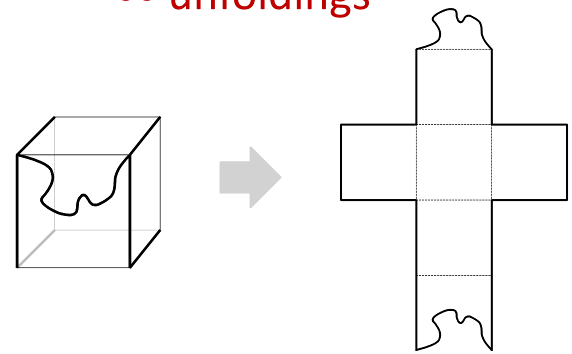
★We consider general unfolding today!

∞ unfoldings



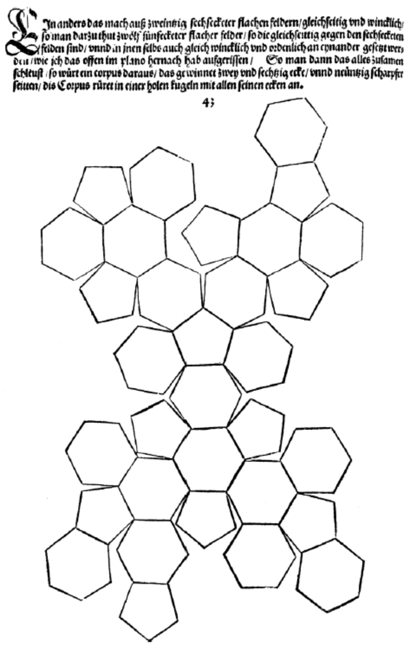


11 edge-unfoldings



Brief history of unfolding

"Underweysung der messing" by Albrecht Dürer (1525)



- He described polyhedra by edge unfolding
- He conjectured the following?

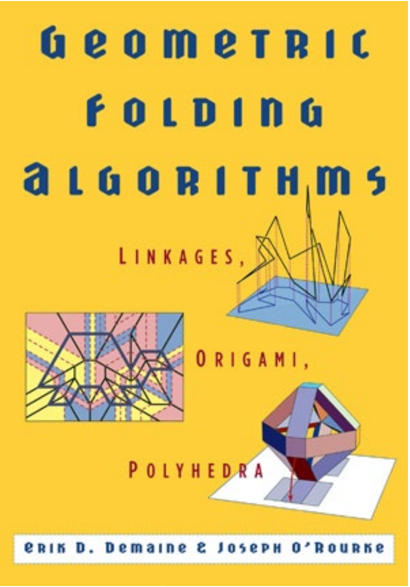
Open Problem:
Any convex polyhedron has an edge unfolding

Brief history of unfolding

Open Problem :
Any convex polyhedron has an edge unfolding

Some related results (which we do not go into):

- (Of course) no counterexamples
- Counterexamples in concave polyhedra
  (Any edge unfolding makes overlapping)
- Possible in general unfolding
  (Cut along shortest paths to all vertices from a general point)
- If you randomly unfold a random convex polyhedron, it causes overlapping with probability almost 1.



If you are interested in....

Summary: We have few knowledge about unfolding

Brief history of unfolding

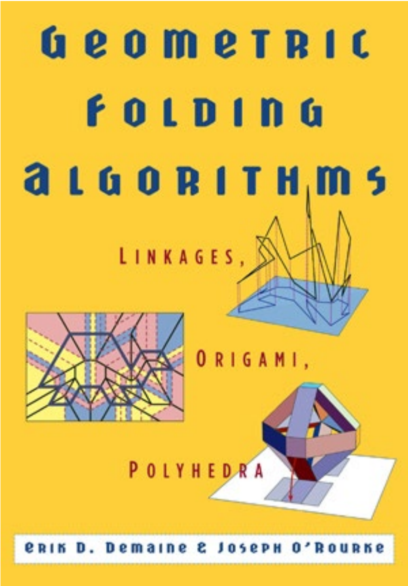
Open Problem：
Any convex polyhedron has an edge unfolding

Summary:

We have few knowledge about unfolding

Main Target in this context:

- For a given simple polygon P, what kind of (convex) polyhedra Q folded from P? (Algorithm/Mathematical Characterization)
- For a given (convex) polyhedron Q, what kind of simple polygons P obtained by unfolding of Q? (Algorithm/Mathematical Characterization)



If you are interested in....

Brief history of unfolding

Summary:

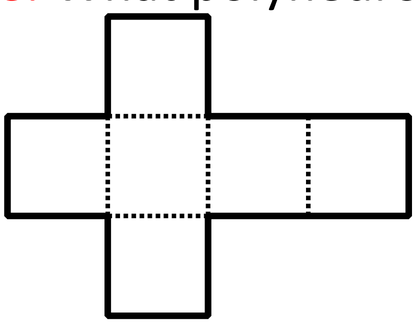
We have few knowledge about unfolding

Main Target in this context:

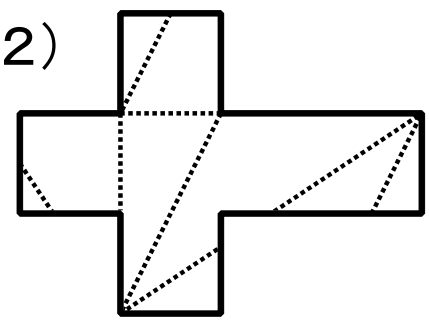
- For a given simple polygon P, what kind of (convex) polyhedra Q folded from P? (Algorithm/Mathematical Characterization)
- For a given (convex) polyhedron Q, what kind of simple polygons P obtained by unfolding of Q? (Algorithm/Mathematical Characterization)

Exercise: What polyhedron you can fold?

（1）



（2）

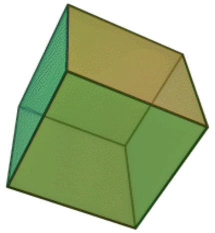


From this "Latin Cross", you can fold 23 different convex polyhedra in 85 different ways!

From this "Latin Cross", you can fold ∞ different concave polyhedra! [found by Uehara, 2014]

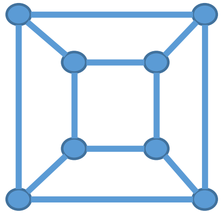
1. Basic of unfolding (1)

Let G be the graph induced by the vertices and edges of a convex polyhedron Q.



[Spanning tree theorem (1)]
Any set of cut lines of Q for an edge unfolding is a spanning tree of G.

Corollary:
On a regular polyhedron, the total cut length of any edge-unfolding is the same.

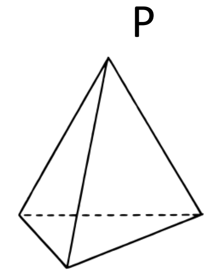


[Proof]

- It visits every vertex of Q:
  Otherwise, we cannot “lay” on a plane.
- No cycle:
  If you have a cycle, the unfolding is disconnected.

[Spanning tree theorem (2)]
Any set of cut lines of Q for a general unfolding is a tree spanning all vertices of Q.

1. Basic of unfolding (2)

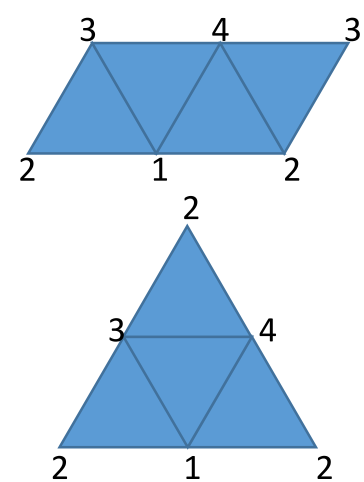


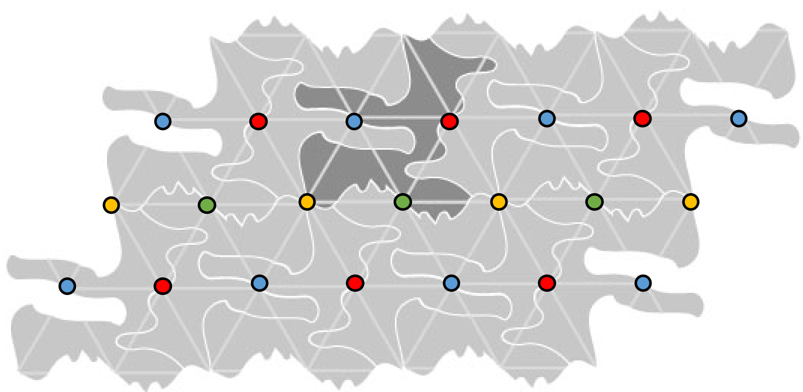
Mathematical characterization of (general) unfolding of a regular tetrahedron

[Unfolding Theorem of a Regular Tetrahedron(Akiyama 2007)]
A polygon P is an unfolding of a regular tetrahedron if and only if it is a tiling satisfying the following conditions:

(1) **P** is a **p2** tiling. (It can "tile" by 180° rotations)
(2) **4 rotation centers** induce a regular triangular lattice
(3) These 4 centers are not "equivalent" on this tiling

Cf: A regular tetrahedron has 2 edge unfoldings

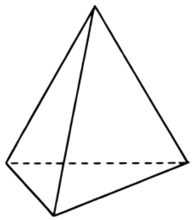




1. Basic of unfolding (2)

Mathematical characterization of (general) unfolding of a regular tetrahedron

P



[Unfolding Theorem of a Regular Tetrahedron(Akiyama 2007)]
A polygon P is an unfolding of a regular tetrahedron if and only if it is a tiling satisfying the following conditions:

(1) **P** is a **p2** tiling. (It can "tile" by 180° rotations)
(2) **4 rotation centers** induce a regular triangular lattice
(3) These 4 centers are not "equivalent" on this tiling

Tile-Makers and Semi-Tile Makers, Jin Akiyama, *The Mathematical Association of America*, Monthly 114, pp. 602-609, 2007.

[Intuitive explanation (not proof)]
If you "roll" a regular tetrahedron 4 times in a proper way, it will return to the original position in original direction.
So if you put ink on it, you can fill the plane by this "stamping".

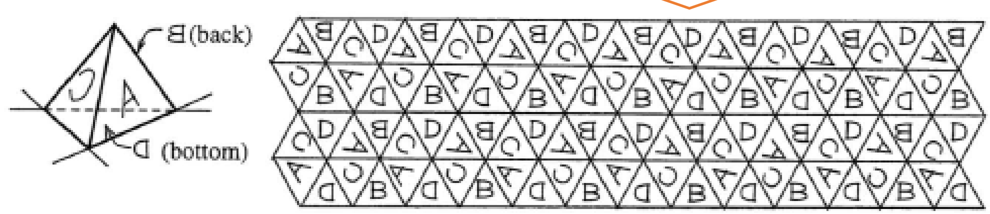
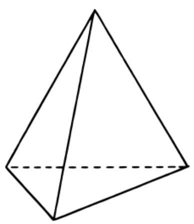


Figure 2.1. Carved regular tetrahedron R and the tiling by stamping with *R*.

1. Basic of unfolding (3)

Tetramonohedron is a tetrahedron consists of 4 congruent acute triangles



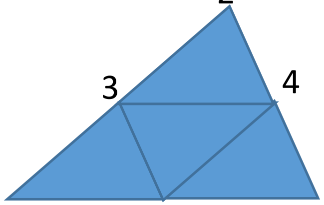
Mathematical characterization of (general) unfolding of a tetramonohedron

[Unfolding Theorem of a Tetramonohedron(Akiyama and Nara 2007)]
A polygon P is an unfolding of a tetramonohedron if it is a tiling satisfying the following conditions:

(1) **P** is a **p2** tiling. (It can "tile" by 180° rotations)
(2) **4** **rotation centers** induce a triangular lattice (by the triangle)
(3) These 4 centers are not "equivalent" on this tiling

[Intuitive explanation (not proof)]
You can "bend" the triangular lattice in the previous theorem for a regular tetrahedron.

Exercise: You can fold a tetramonohedron from any acute triangle. What happens for an obtuse triangle?



1. Basic of unfolding: Some Exercises

1. Pick up one, say P, of 11 edge unfolding of a cube. Find as many convex polyhedra folded from P as you can find. What can you say the conditions that you can obtain a convex polyhedron from P?
2. Show that you can fold to a tetramonohedron from any acute triangle. What happens you try to fold from an obtuse triangle? Consider and discuss convex and concave quadrilaterals.
3. Find the shortest cut length of regular polyhedra.
   - For a regular tetrahedron, we have a beautiful solution.
     - Show the optimal solution and proof (if possible)
   - For a regular octahedron and a cube;
     - You may find optimal solutions,
     - But showing the optimality is tough…
   - For a regular icosahedron and a dodecahedron;
     - Finding optimal solutions may be tough?

Computational ORIGAMI=

Geometry＋Algorithm＋Computation

- Mathematics
- Theoretical Computer Science
- Real High Performance Computing

- Many Applications from micro-size to universe-size
  - Bioinfomatics (e.g., DNA folding),
  - Robotics, packaging,
  - Architecture

- Many young researchers;
  - even undergrad students, highschool students!