Computational Origami

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- Schedule
  - January 27 (13:30-15:10)
    - Introduction to Computational Origami
    - Polygons and Polyhedra folded from them
  - January 29 (10:50-12:30)
    - Computational Complexity of Origami algorithms
  - February 3 (9:00-10:40)
    - Advanced topics
    - 13:30-15:10 (Office Hour at I67-b)

- Report (up to 20pts)
  - Submit a report about one of the following two options:
  1. Survey some paper(s) appearing in these three lectures
  2. Solve some problems appearing in these three lectures

  - Firm deadline: 17:00, February 10 in one of the following two ways
    - By email:
      PDF file (word file is not acceptable) from JAIST account.
    - By paper:
      A4 size paper, staple at the top-left corner.

    You can write your report in English or Japanese.

Topics in the lectures

Computational Geometry

Part 1: Polygons and polyhedra folded from them

- Relationship between unfolding and solids: Big open problem
- How can we compute (convex) “polyhedra” from a given “unfolding”?
  - Mathematical characterization/algorithms/computation power

Algorithms and Computational Complexity

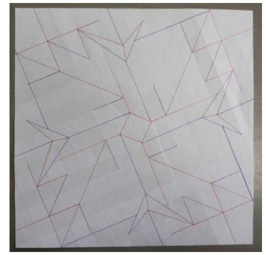
Part 2: Algorithms and computational complexity of “folding”

- Basic operations of origami
- Algorithms and complexity of origami
  - Efficiency of folding of 1-dimensional origami (algorithms and complexity)
    - Efficient algorithm (how can we reduce the number of folding?)
    - How can we evaluate “good” folded states?

There are many open problems, where many young researchers working on

I have not yet reach to 2-dimension…

History…



Kawasaki Rose

- June 22, 2008

at “Forth Origami in Science, Mathematics, and Education”,

Professor Toshikazu Kawasaki, who is a mathematician and the designer of Kawasaki Rose, said that:

“As a mathematician, I do not take care once if we can show the existence of solutions.”

Trivia: The other twin Kawasaki is a professor of OR in Kyushu University.

As a computer scientist, I considered…

It is important that how can we obtain the solution, and It’s computational cost

- GOOD algorithms?
- Computational complexity

We need good problem…

Objective: Estimate "Complexity" of origami design

I would like to give the answer to the question "which is more complex?"

Kawasaki Rose

You need to fold in 3 dimensional space. For Uehara, it took 10 days at first, but now it takes about 10 minutes.

For Uehara, it took 40 minutes, and even Maekawa-san needs 20 minutes to fold.

Maekawa Devil

Both are more complicated than the classic "crane", but why can you say that?

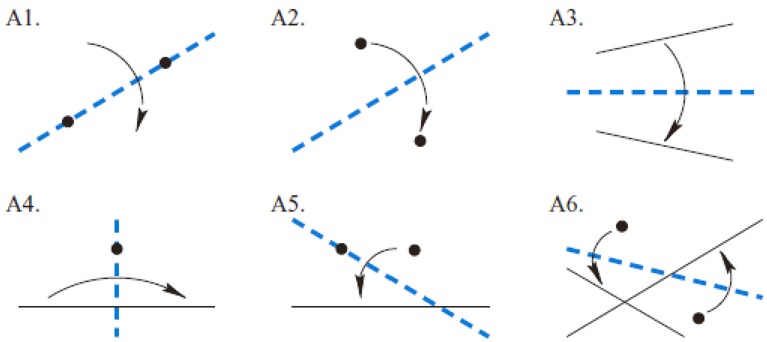
Computational complexity and algorithm theory

- Basic theory of theoretical computer science
  - Basic machine models:

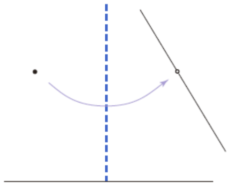
    Common agreement of "basic operations"
    - Turing machine
    - VRAM model
  - Algorithm = sequence of basic operations
    - How can we combine basic operations?
  - Complexity of algorithm
    - Time complexity: Estimate by the number of basic operations
    - Space complexity: Estimate by the quantity of memory cells

“Complexity” of folding…?

- Basic origami operations:
  - In origami society, they agree that the following seven operations as “basic operations”
  - We consider them as “basic operations”



Huzita’s axioms A1-A6



Hatori’s operation A7

These 7 operations can solve up to quartic (4) equations. (Traditional “ruler+compass” can solve up to quadratic (2) equations.)

“Complexity” of folding…?

- “Reasonable” origami models to investigate computational complexity? (Simple → Complex)
  1. The simplest model: 1D, creases in unit distance
     - On a long paper strip, we fold along vertical crease lines of unit interval
  2. 2 directions to extend
     - Crease lines in non-unit interval
       - (Crease lines can be *slanted*)
     - Higher dimensions (2 or 3 or more?)

We are still around here…

Complexity/Efficiency of Origami(?)

- From the viewpoint of Theoretical Computer Science…
- Two **resources** of Turing machine model
  1. Time: The number of basic operations
  2. Space: The quantity of memory cells

Complexity/Efficiency of Origami(?)

- From the viewpoint of Theoretical Computer Science…
- Two **resources** of Origami model
  1. Time: The number of basic (folding) operations
     - J. Cardinal, E. D. Demaine, M. L. Demaine, S. Imahori, T. Ito, M. Kiyomi, S. Langerman, R. Uehara, and T. Uno: Algorithmic Folding Complexity, *Graphs and Combinatorics*, Vol. 27, pp. 341-351, 2011.
     - (Guoxin Hu, Shin-Ichi Nakano, Ryuhei Uehara and Takeaki Uno: Simple Fold and Cut Problem for Line Segments, *CCCG 2019*, pp. 158-163, 2019/08/08-10, Edmonton, Canada.)
  2. Space: The quantity of ??
     - R. Uehara: On Stretch Minimization Problem on Unit Strip Paper, *22nd Canadian Conference on Computational Geometry*, pp. 223-226, 2010/8/9-11.
     - Takuya Umesato, Toshiki Saitoh, Ryuhei Uehara, and Hiro Ito: Complexity of the stamp folding problem, *5th Annual International Conference on Combinatorial Optimization and Applications (COCOA '11)*, Lecture Notes in Computer Science, Vol. 6831, pp. 311-321, 2011/8/4-6, Zhangjiajie, China.
     - Erik D. Demaine, David Eppstein, Adam Hesterberg, Hiro Ito, Anna Lubiw, Ryuhei Uehara and Yushi Uno: Folding a Paper Strip to Minimize Thickness, *The 9th Workshop on Algorithms and Computation (WALCOM 2015)*, Lecture Notes in Computer Science Vol. 8973, pp. 113-124, 2015/02/26-2015/02/28, Dhaka, Bangladesh.

Today's Topic

The 7th EATCS/LA Presentation Award!

5. Time Complexity
    - "Folding complexity"
        - Theoretically, the world fastest algorithm for pleat folding
6. Space Complexity (?)
    - Stamp Folding Problem
    - Minimization of Crease width
        - NP-complete problem, FPT algorithm
7. Undecidable Origami Problem
    - Diagonalization and undecidability

On the other hand; This paper was rejected by some journal because "the proof is too simple", which is an important point in the paper ;-)
Sometimes, it is difficult to propose new computation model…

March 2012
IPSJ Yamashita SIG Research Award

Exercise (or challenge)

1. Propose some measurement that evaluate "complexity" of origami
   - Example: space for folding
2. Discuss about the measurement
   - Example: For one dimensional origami, the space for folding can be important. Imagine folding a long, say, 1km, metal pipe for water supply. On the other hand, in two dimensional space, Origami (in square) basically shrinks in[illegible]h folding usually, so it is not reasonable.

Goal: In Turing machine, we have "time-space trade off". It is quite nice to have such a measure that has a kind of trade off property between the number of folding.