Computational Origami

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- Schedule
  - January 27 (13:30-15:10)
    - Introduction to Computational Origami
    - Polygons and Polyhedra folded from them
  - January 29 (10:50-12:30)
    - Computational Complexity of Origami algorithms
  - February 3 (9:00-10:40)
    - Advanced topics
      1. (Bumpy) Pyramid Folding
      2. Zipper Unfoldability
    - 13:30-15:10 (Office Hour at I67-b)

Zipper Unfolding of Domes and Prismoids

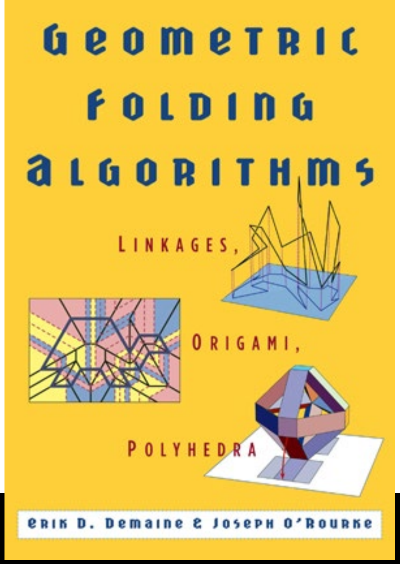
Reference:

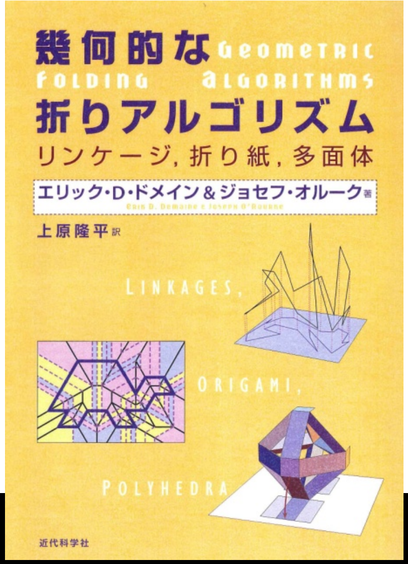
Erik D. Demaine, Martin L. Demaine and Ryuhei Uehara: Zipper Unfoldability of Domes and Prismoids, *The 25th Canadian Conference on Computational Geometry (CCCG 2013)*, pp. 43-48, 2013/08/08-2013/08/10, Waterloo, Canada.

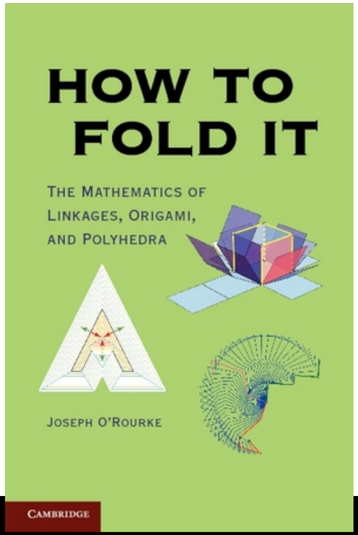
Since Dűrer (1525), ...

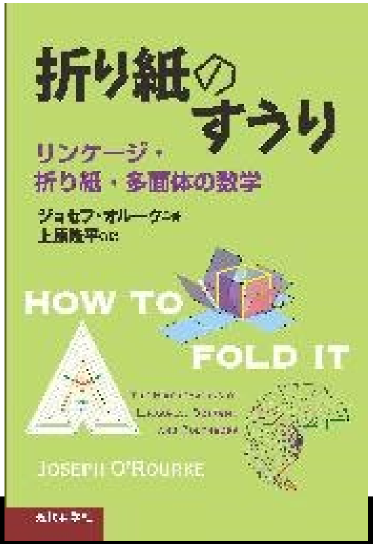
Big Open Problem in Computational Origami:

Any convex polyhedron can be developed into a flat (nonoverlapping) polygonal shape by cutting only along its edges?









Since Dűrer (1525), …

Big Open Problem in Computational Origami:

In my personal sense, the difficulty comes from

1. "edge cutting": combinatorics of spanning trees on a graph (induced by the edges)
   (cf. any development forms a spanning tree)
   - if you allow to cut inside of faces, you can get a nonoverlapping development.
2. "overlapping": geometry of the shape of the polyhedron
   - if you don't mind overlapping, any spanning tree gives you a "development."

Since Dűrer (1525), …

Big Open Problem in Computational Origami:

We concentrate on "overlapping" with restricted "edge cutting":

"Edge cutting" should be Hamiltonian path;

Hamiltonian unfolding investigated in [Shephard 1975][Demaine^2, Lubiw, Shallit^2 2010]

Applications:

Since Dűrer (1525), …

Big Open Problem in Computational Origami:

We concentrate on “overlapping” with restricted “edge cutting”:

“Edge cutting” should be Hamiltonian path;

Hamiltonian unfolding investigated in [Shephard 1975][Demaine^2, Lubiw, Shallit^2 2010]

Applications:

Hamiltonian unfolding

Known results [DDLSS 2010]:

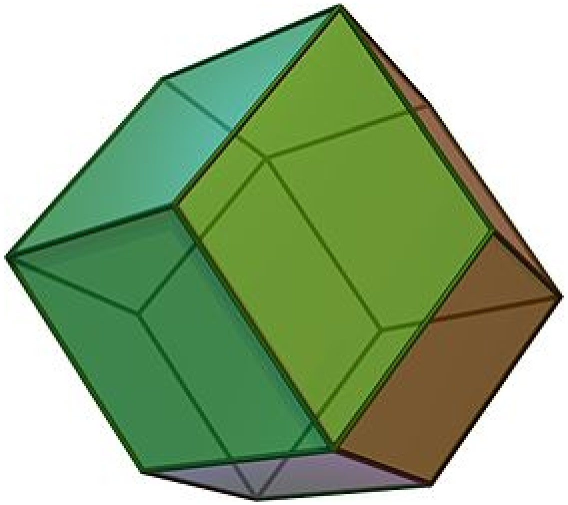
Most regular convex polyhedra are Hamiltonian unfoldable.

But, e.g., a rhombic dodecahedron

has no Hamiltonian unfolding

because it has

no Hamiltonian path.



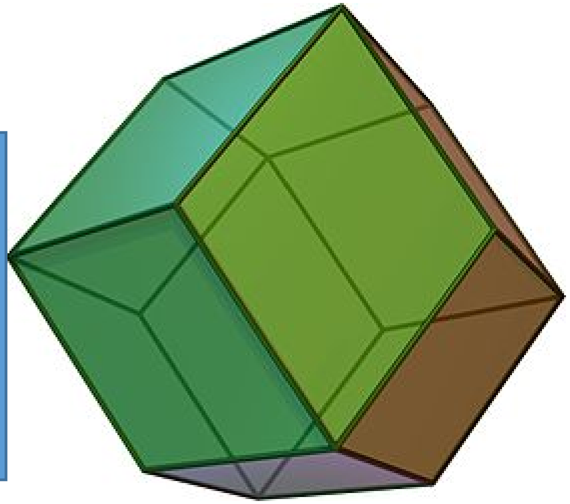
Hamiltonian unfolding

A rhombic dodecahedron has no Hamiltonian unfolding since it has no Hamiltonian path.

We introduce/consider Hamiltonian unfolding to concentrate on overlapping!?

Natural question:

Are there (natural) polyhedra that
1. have many Hamiltonian paths
2. Hamiltonian ununfoldable because of overlapping?



Hamiltonian unfolding

Main results:

1. Sequence of *domes* that
   1. have many Hamiltonian paths
   2. Hamiltonian ununfoldable because of overlapping!
2. Hamiltonian unfoldability of *any nested prismoid*
3. Poly-time algorithm for Hamiltonian unfoldability of *any (general) prismoid*

Hamiltonian unfolding: ununfoldable domes

Dome: convex polyhedron that consists of

- single *base* (convex *n*-gon)
- *n* sides (convex polygon)

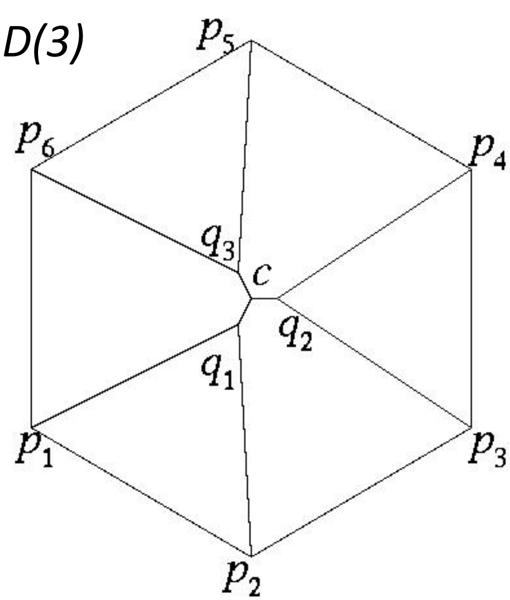
- E.g. dome with pentagon base with 5 sides

Th 2: Hamiltonian-ununfoldable domes

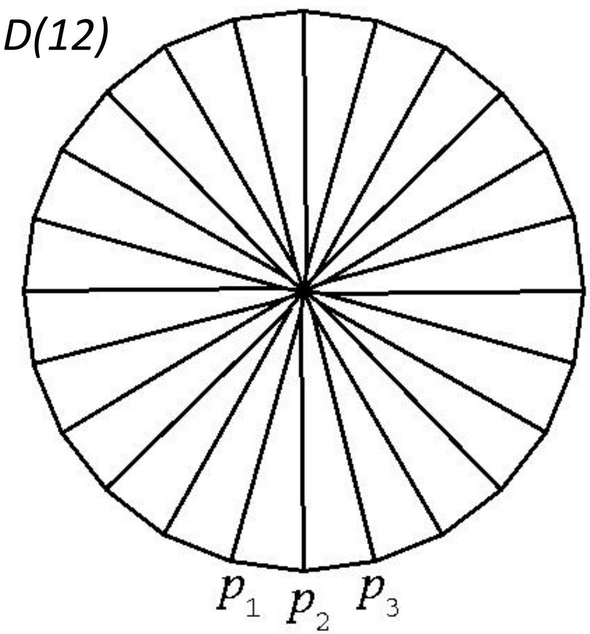
Th 2: There exists an infinite sequence of Hamiltonian ununfoldable domes.

[Proof] Constructive. For the following *D(i)*, we have the theorem for $i \geqq 12$.

D(3)



D(12)

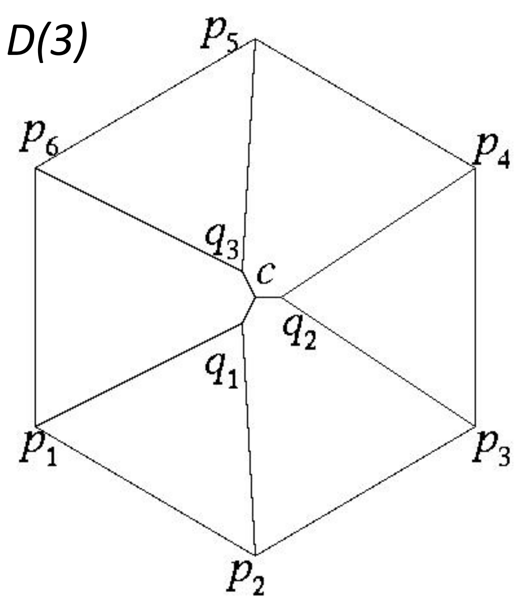


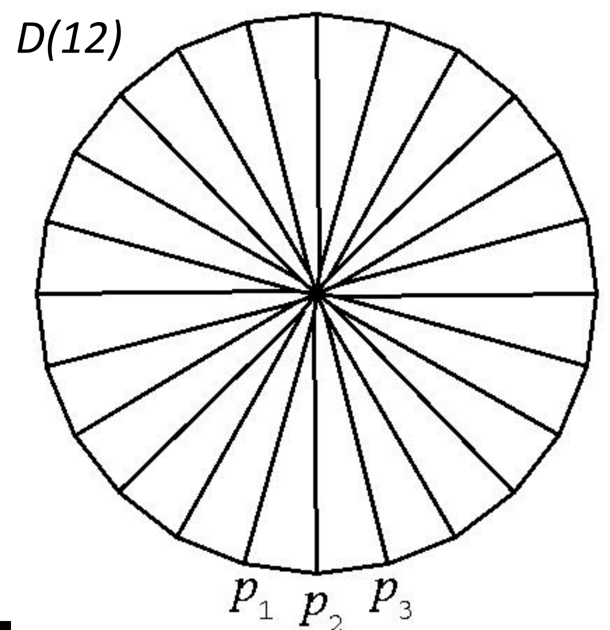
Th 2: Hamiltonian-ununfoldable domes

[Obs 1] We have many Hamiltonian-paths P.

[Obs 2] deg(c)=1 or 2 on any HP P.

[Obs 3] Most q_i ($\leqq$2 exceptions) are tops of flap.





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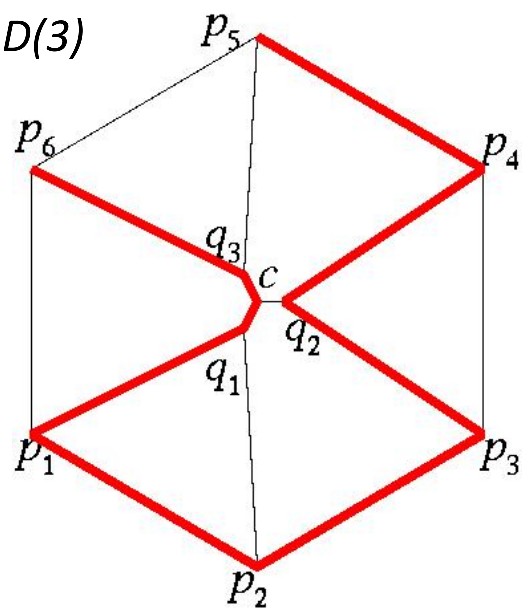

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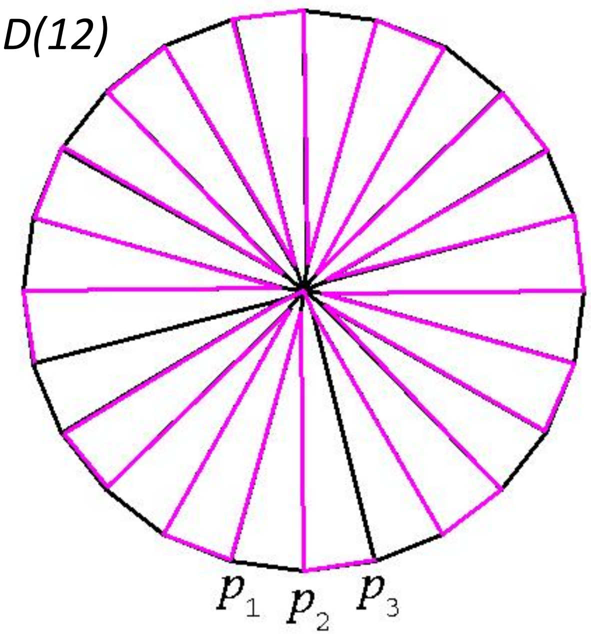


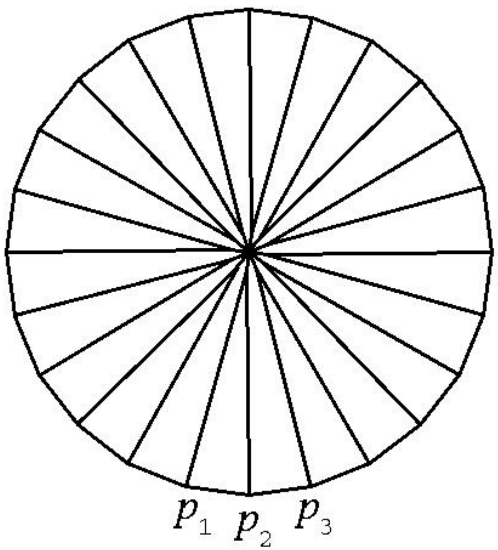
Th 2: Hamiltonian-ununfoldable domes

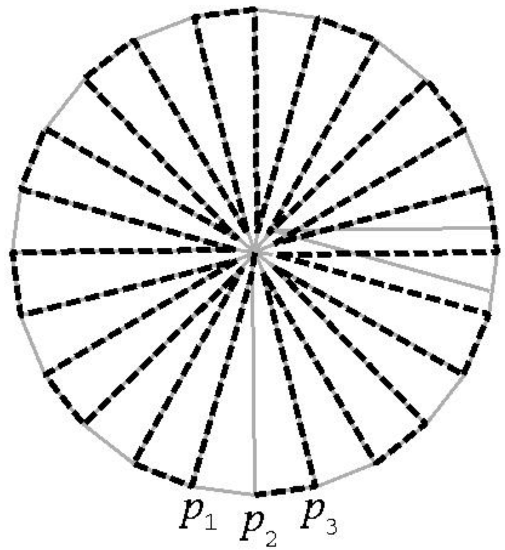
[Obs 1] We have many Hamiltonian-path *P*.

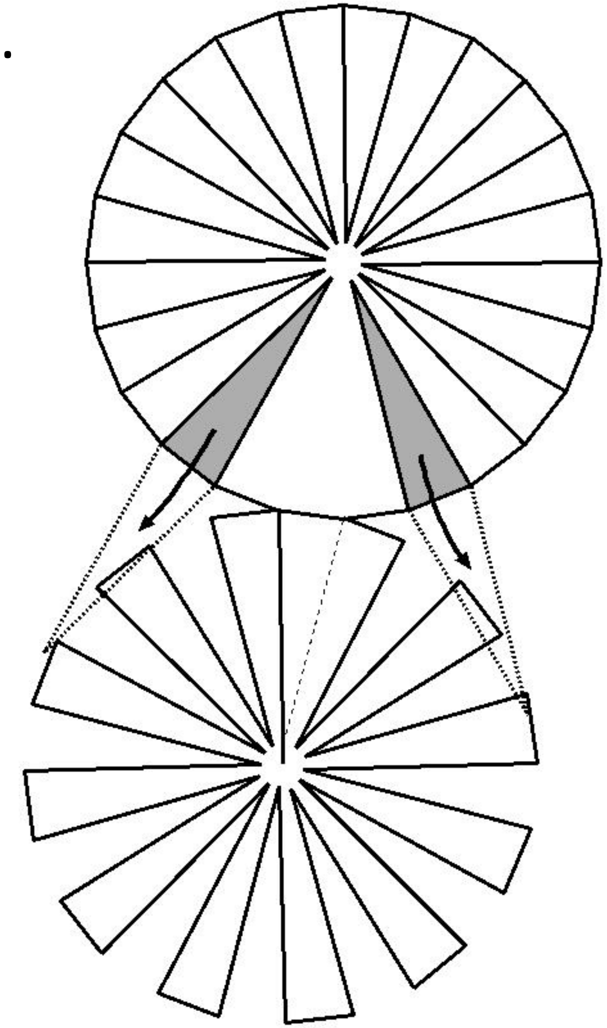
[Obs 2] deg(*c*)=1 or 2 on any HP *P*.

[Obs 3] Most q_i are tops of flap.

Overlap should occur for any $i \geqq 12$!







Hamiltonian unfolding

Main results:

1. Sequence of *domes* that
    1. have many Hamiltonian paths
    2. Hamiltonian ununfoldable because of overlapping!
2. Hamiltonian unfoldability of *any nested prismoid*
3. Poly-time algorithm for Hamiltonian unfoldability of *any (general) prismoid*

Hamiltonian unfolding

Main results:

2. Hamiltonian unfoldability of *any nested prismoid*

   Th. 4: Any nested prismoid has a Hamiltonian unfolding.

3. Poly-time algorithm for Hamiltonian unfoldability of *any (general) prismoid*

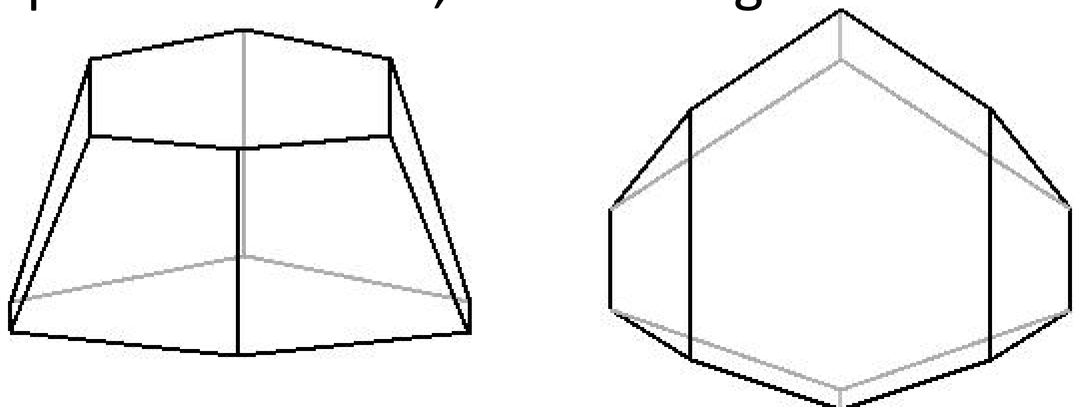
   Th. 5: The number of HPs in a prismoid with n vertices is $O(n^3)$.

Just count it!

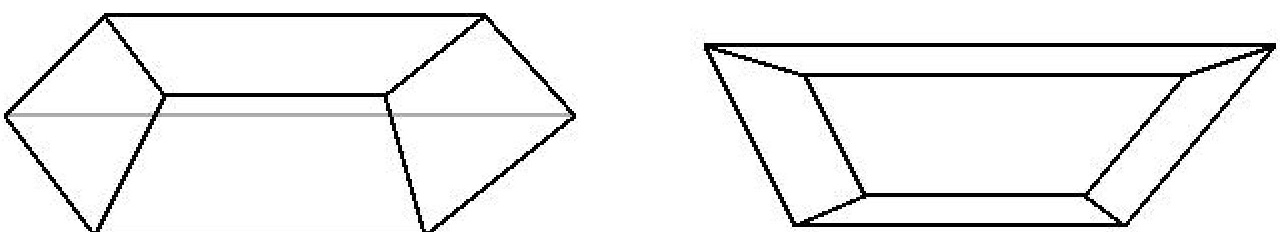
Prismoid

Prismoid: convex hull of two parallel convex polygons with matching angles

- It has “top” and “bottom,” whose angles match.



Nested prismoid: orthogonal projection of the top is included in the bottom.



Hamiltonian unfolding

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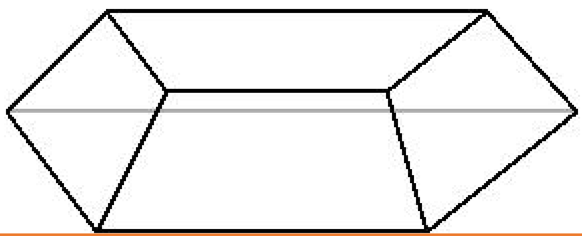

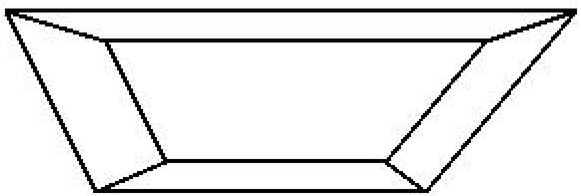

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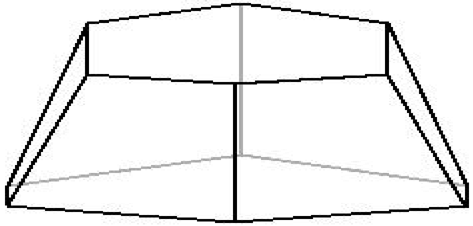
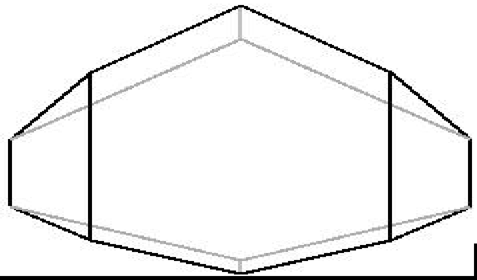


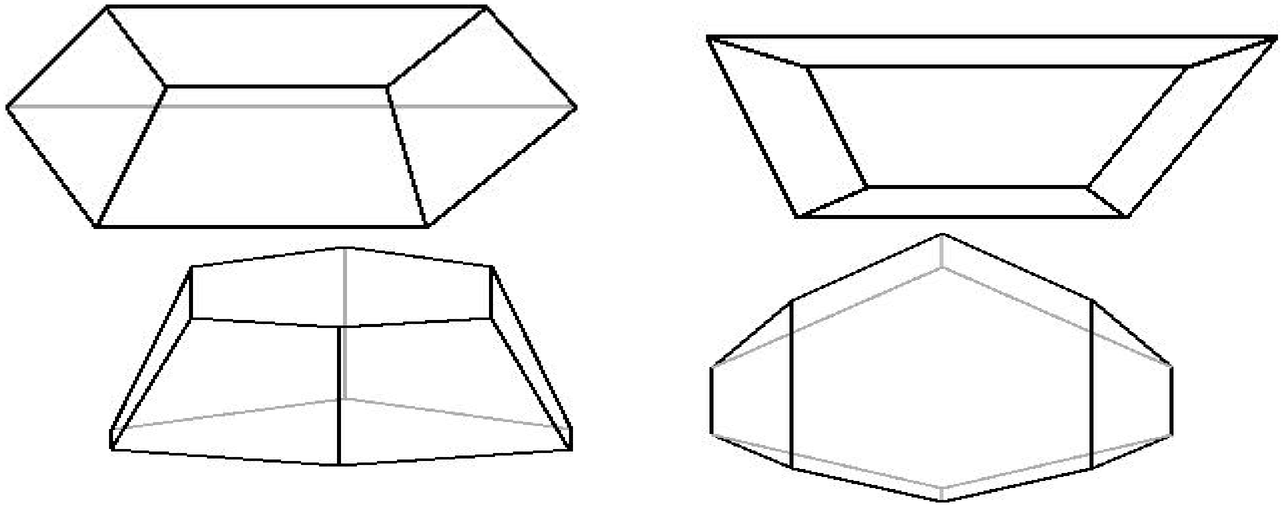


Related results

Band of any prismoid (=without top/bottom) can be unfolded by cutting an edge (not *any* edge).

Nested [Aloupis, et al. 2004/2008]

General [Aloupis 2005](Ph.D Thesis)

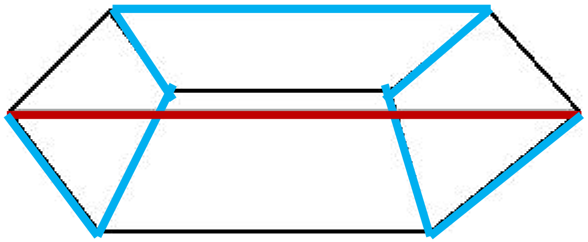


Hamiltonian unfolding

Th. 4: Any *nested* prismoid has a Hamiltonian unfolding.

Basic idea:

1. cut the edge to unfold the band
2. cut its neighbor edge and around top
3. choose suitable edge to attach the band to base
4. cut the remaining edges



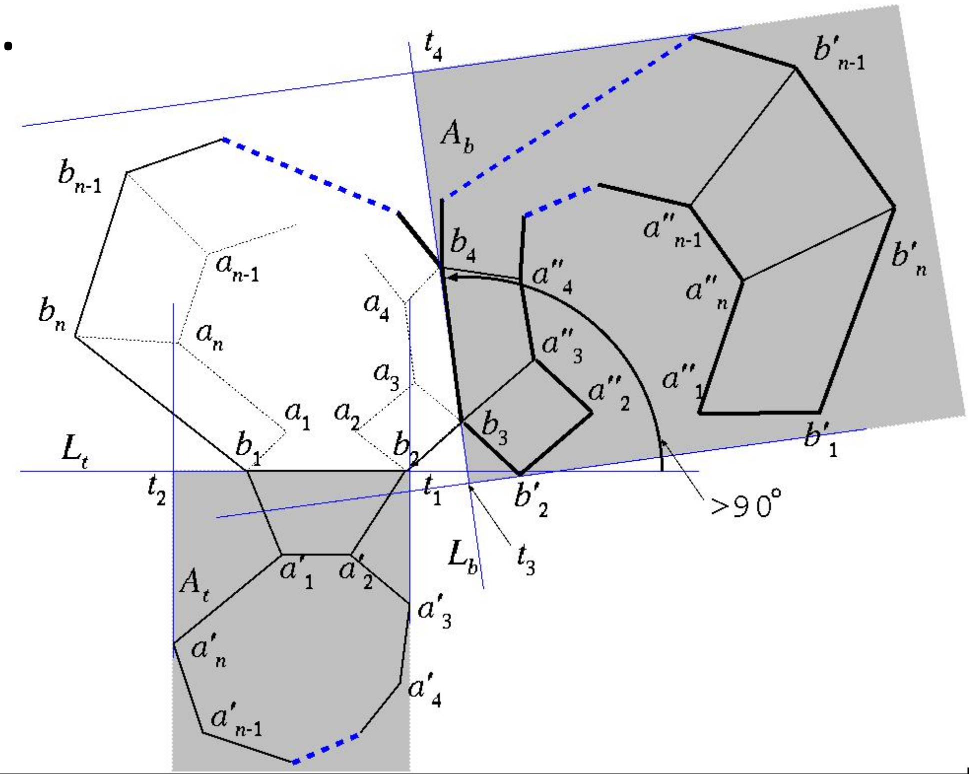
Hamiltonian unfolding

Main results:

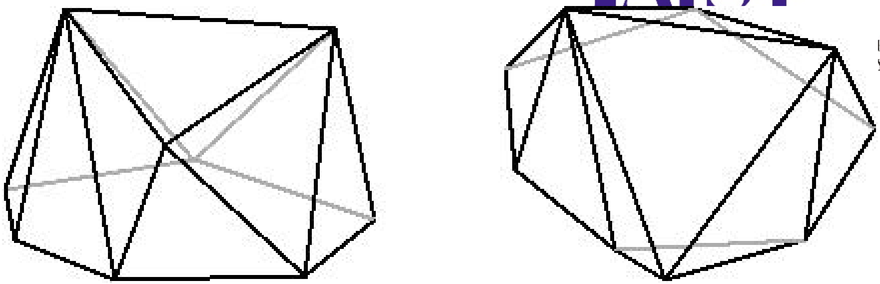
Th. 4: Any *nested* prismoid has a Hamiltonian unfolding.

Q: Does it work for general prismoid?

A: I don’t know, so far.



Conclusion



Conjecture:

Any *general* prismoid has a Hamiltonian unfolding.

(How can we avoid overlap between top and band?)

Nest step:

- Prismatoid: convex hull of two parallel convex polygons
  - they have exponentially many HPs
  - so far, we have no Hamiltonian ununfoldable one (even for bands)

Computational ORIGAMI=

Geometry＋Algorithm＋Computation

- Mathematics
- Theoretical Computer Science
- Real High Performance Computing

- Many Applications from micro-size to universe-size
  - Bioinfomatics (e.g., DNA folding),
  - Robotics, packaging,
  - Architecture

- Many young researchers;
  - even undergrad students, highschool students!