Pattern-Guided Simulations of Immersed Rigid Bodies

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“Immersed body dynamics”

Natural,
Efficient,
Turbulent,
Coupling
simulations in CG

Where we stand?

Unsolved problem

Issac Newton

James Maxwell

Gustav Kirchhoff

James Lighthill

Phenomenal models

(Physics)

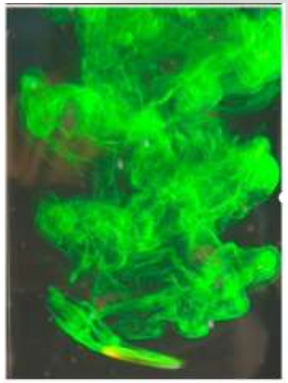
[1964]

[1998]

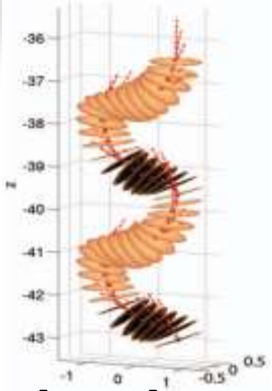
[2005]

[2009]

[2011]



[2013]



[2013]

Simulation models

(Graphics)

[AD,SIG91]

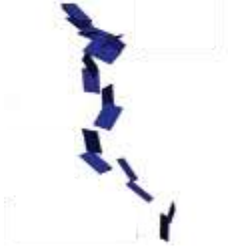
[LBM,TVCG04]

[SPH,TVC05]

[AD,SIG12]

[Turb,SA13]

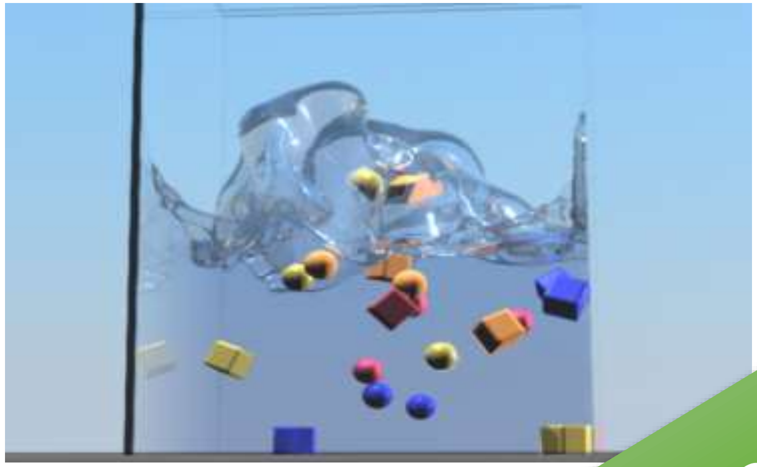
[Data,TVC14]

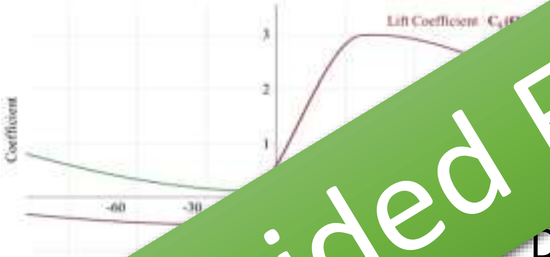


[Data,SIG15]

Challenges

- Coupling (fluid) simulations
  - Not efficient!
- Aerodynamic simulations
  - Not complex!
- Direct motion capture simul
  - Not easy!
- Physics model simulati
  - Not real!



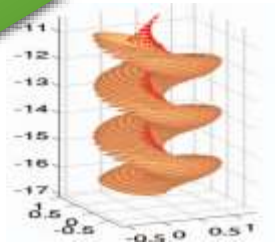


Drag and Lift coefficients, SIG14

Pattern-guided Framework

Heavy markers

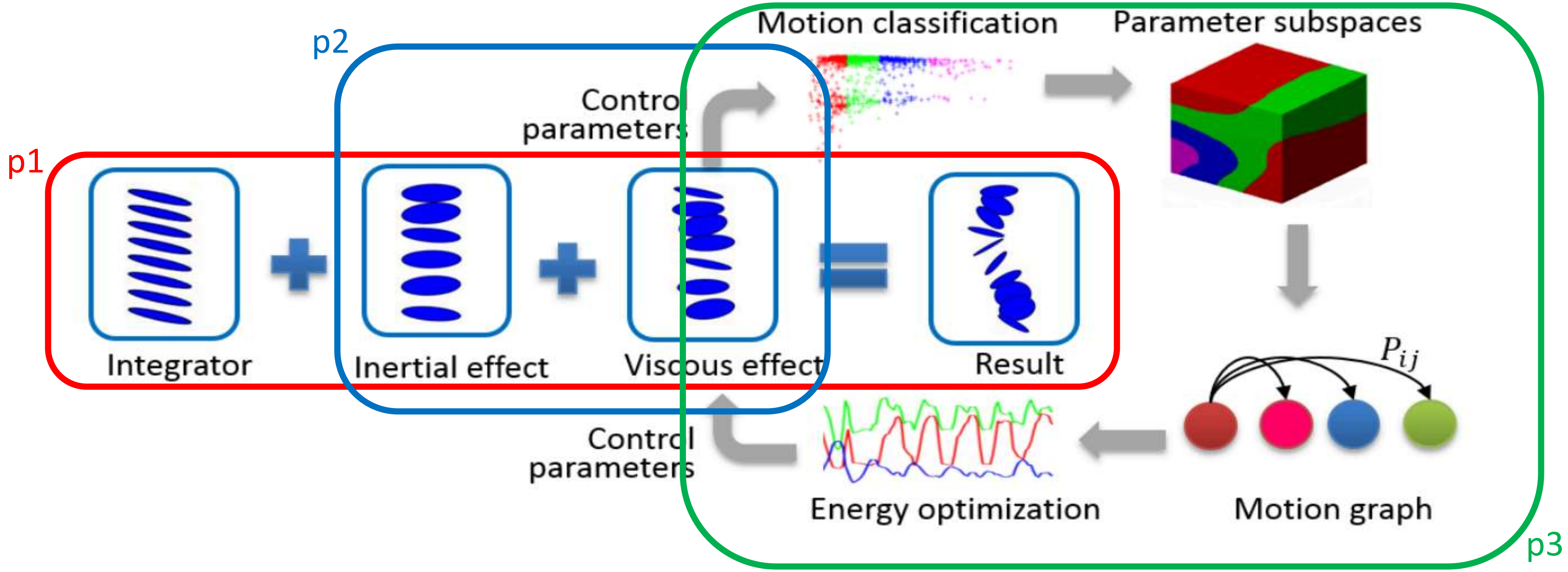
Motion blur



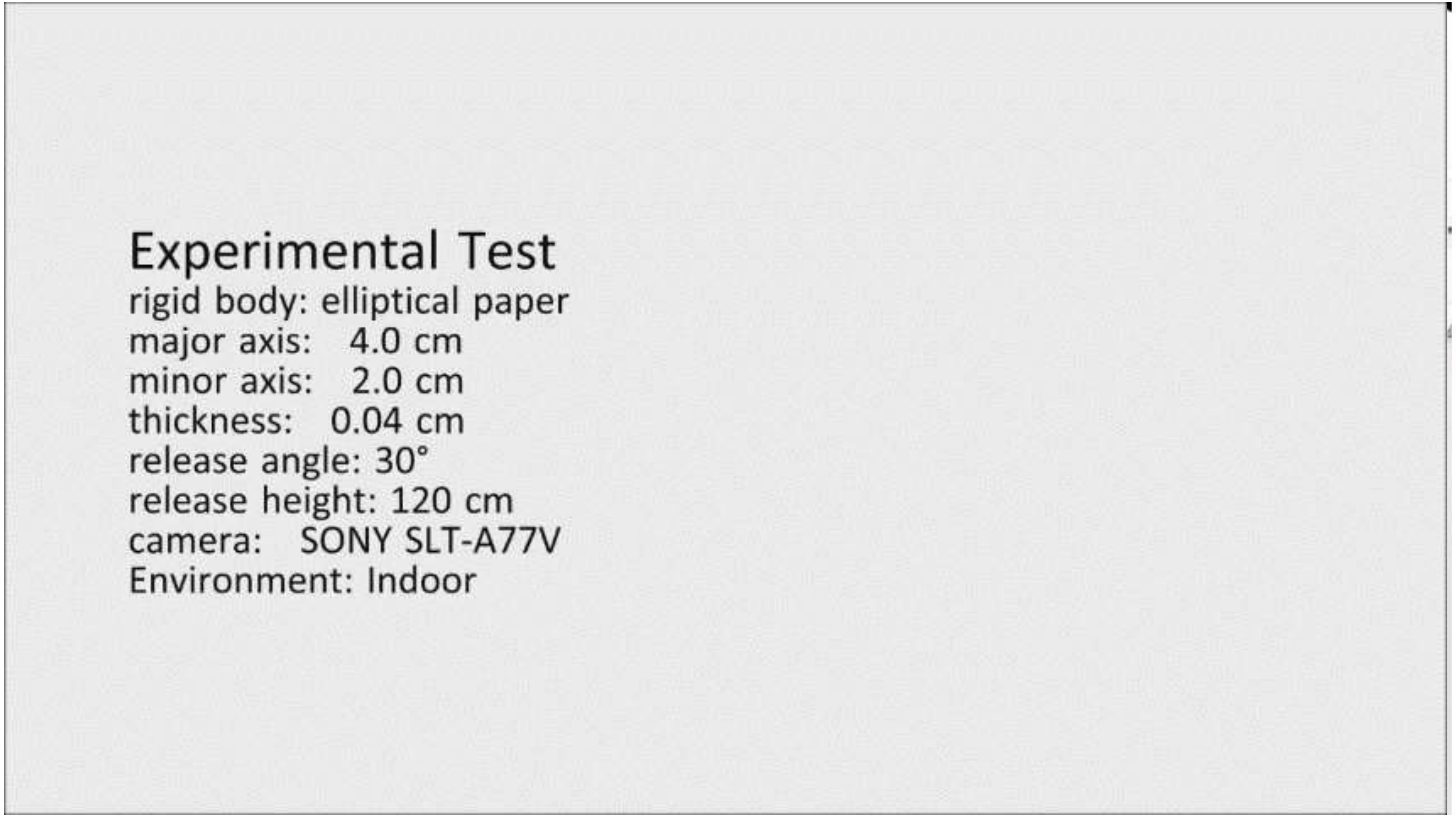
J. Fluid Mech 2013

Pattern-guided Framework —a macro intro

- Principles
  - p1. Avoiding the computations of fluid motions
  - p2. Accounting for the surrounding flow effects
  - p3. Hybrid approach of numerical and data-based methods



What is **Patterns?** Real environments



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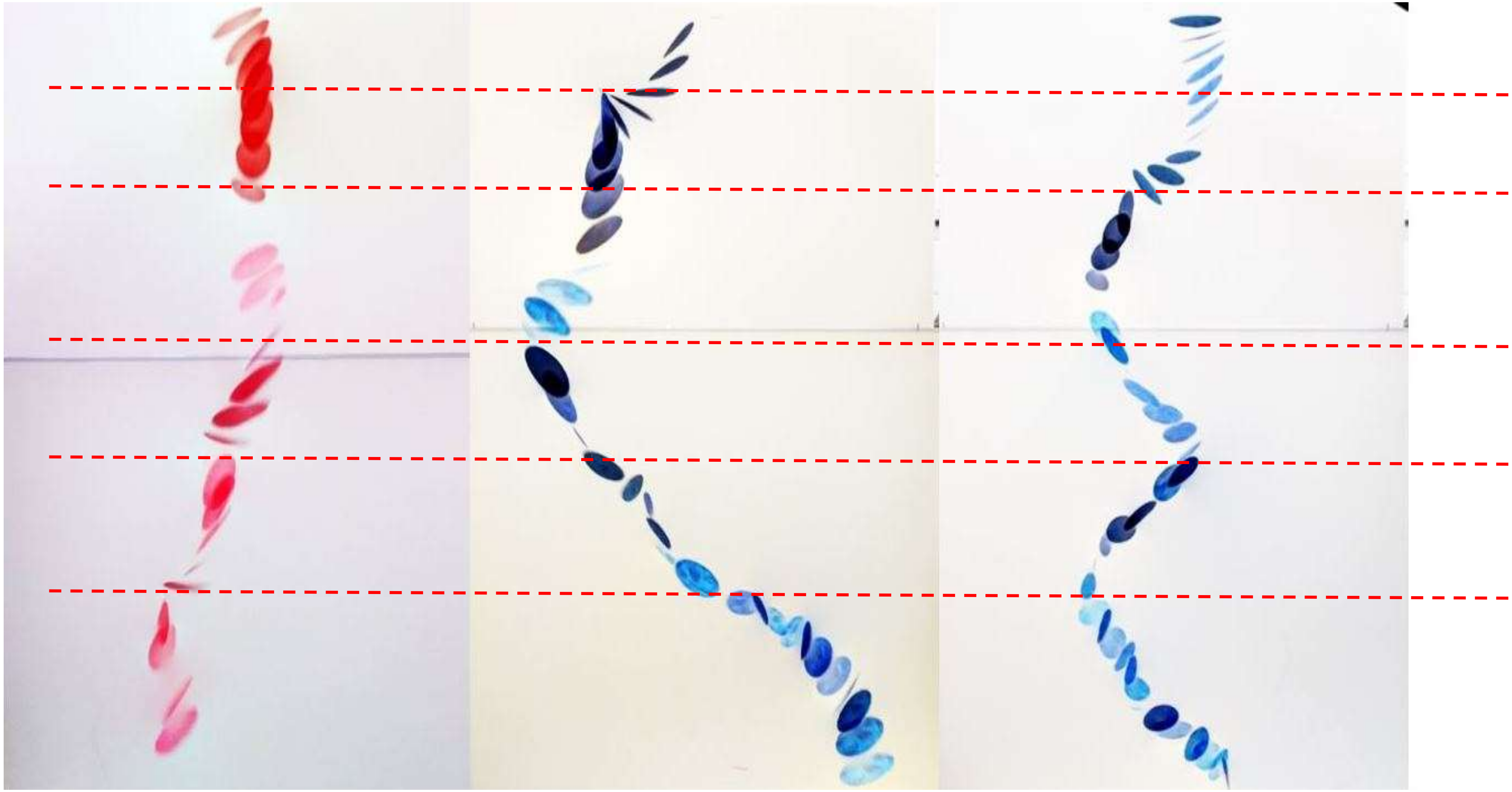


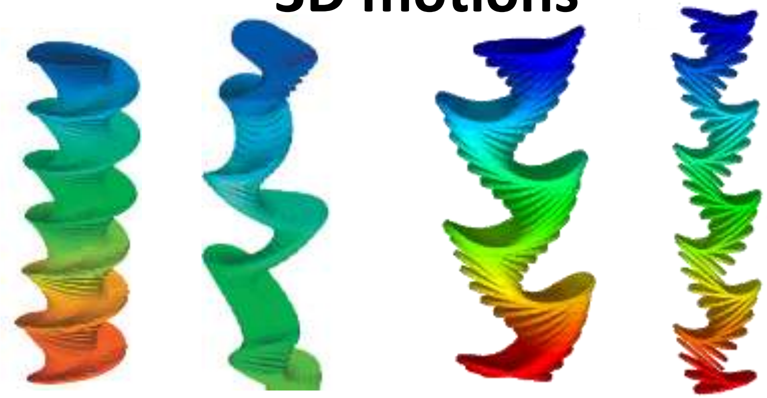
What is **Patterns?**

Ideal environments

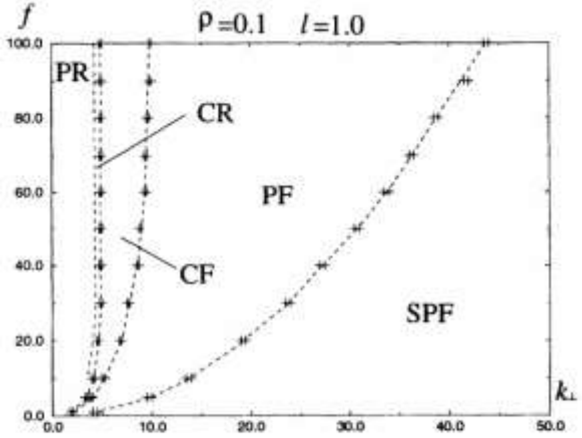
2D motions

[Field, Nature 1997]

3D motions

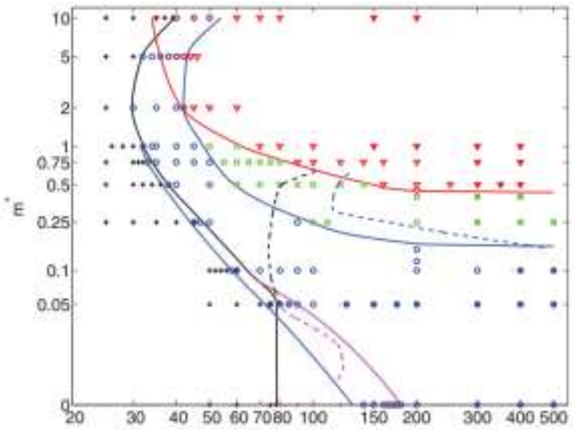


[Zhong, J. Fluid Mechanics 2013]



[Tanabe, Phys. Rev. Lett. 1994]

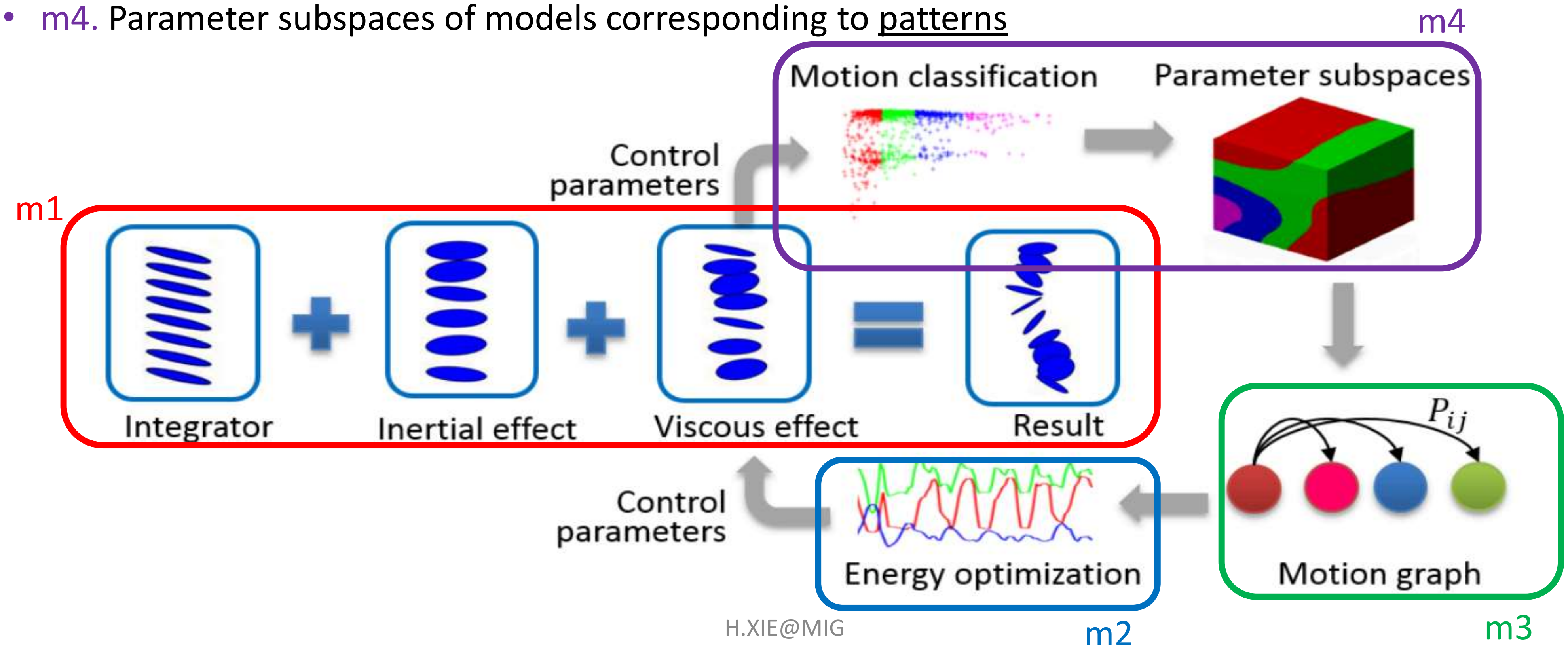
Phase diagram

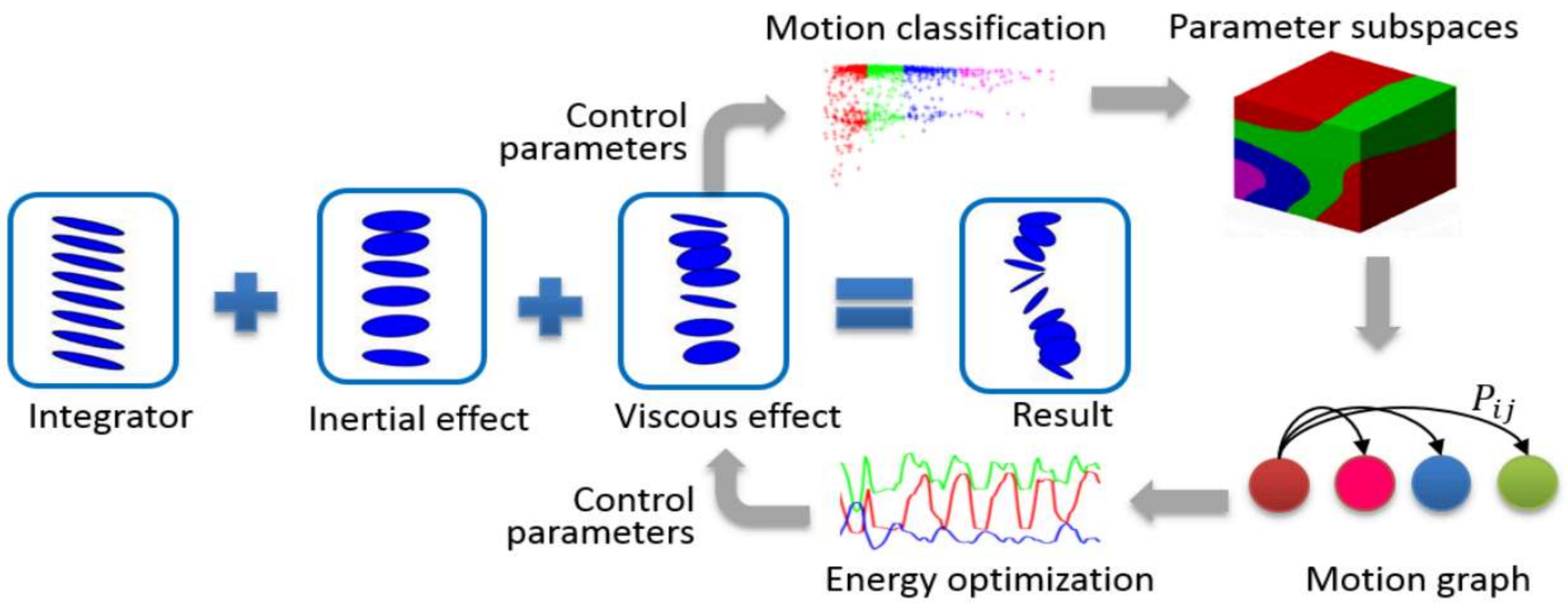


[Chrust, Physics of Fluids 2013]

Pattern-guided Framework —a micro intro

- Methodology
  - m1. Ideal motions are simple, but primitive patterns
  - m2. Motion transitions among patterns at turning points
  - m3. Motion capture of motion patterns, better than trajectories
  - m4. Parameter subspaces of models corresponding to patterns





Flow Effects

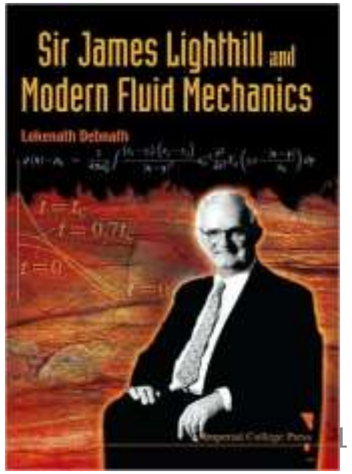
F1. Inertial effect,

F2. Viscous effect,

F3. Turbulent effect

…the **force on a body may be divided into (i) a potential-**flow force **that depends linearly on the body velocity, and can be accurately calculated; and (ii) a vortex-**flow force **that varies nonlinearly and is related in a de**finite way to vortex **shedding and to the convection of shed vorticity.**

--Sir James Lighthill



F1. Inertial effect

Dynamical Model

Laplace Equations:

$$\begin{aligned} \Delta\phi(z) &= 0 \quad z \in \Omega \\ \nabla\phi(z) \cdot n &= u_n(z) \quad z \in \partial B \\ \phi(z) &= 0 \quad \|z\| \to \infty \end{aligned}$$

: potential field

Kinematic Equations:

$$\begin{pmatrix} \dot{R} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} R\hat{\omega} \\ Ru \end{pmatrix}$$

Dynamical Equations:

$$M_a \frac{du}{dt} = (M_a u) \times \omega$$

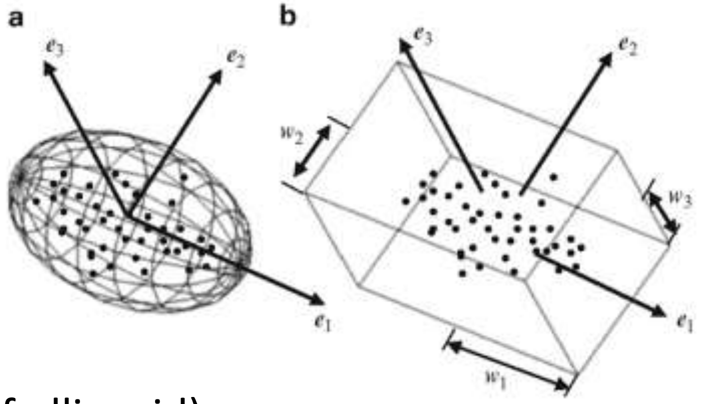
Added mass

$$I_a \frac{d\omega}{dt} = (I_a \omega) \times \omega + (Mu) \times u$$

Added moment of inertia

Analytic Added Tensors

- A simple and efficient approach for Laplace Equations
  - Bounding ellipsoid
  - Approximated solution



Kinetic Energy(t): $$E_a = \frac{1}{2} m_f \frac{a_0 u^2}{2 - a_0}$$ (a,b,c: axes of ellipsoid)

$$a_0 \equiv abc \int_0^\infty \frac{d\lambda}{(a^2+\lambda)\sigma}$$

$$\sigma = \sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)}$$

Kinetic Energy(r): $$E_a^r = \frac{1}{2} m_f \frac{(b^2-c^2)^2(c_0-b_0)\omega^2}{10(b^2-c^2) - 5(b^2+c^2)^2(c_0-b_0)}$$

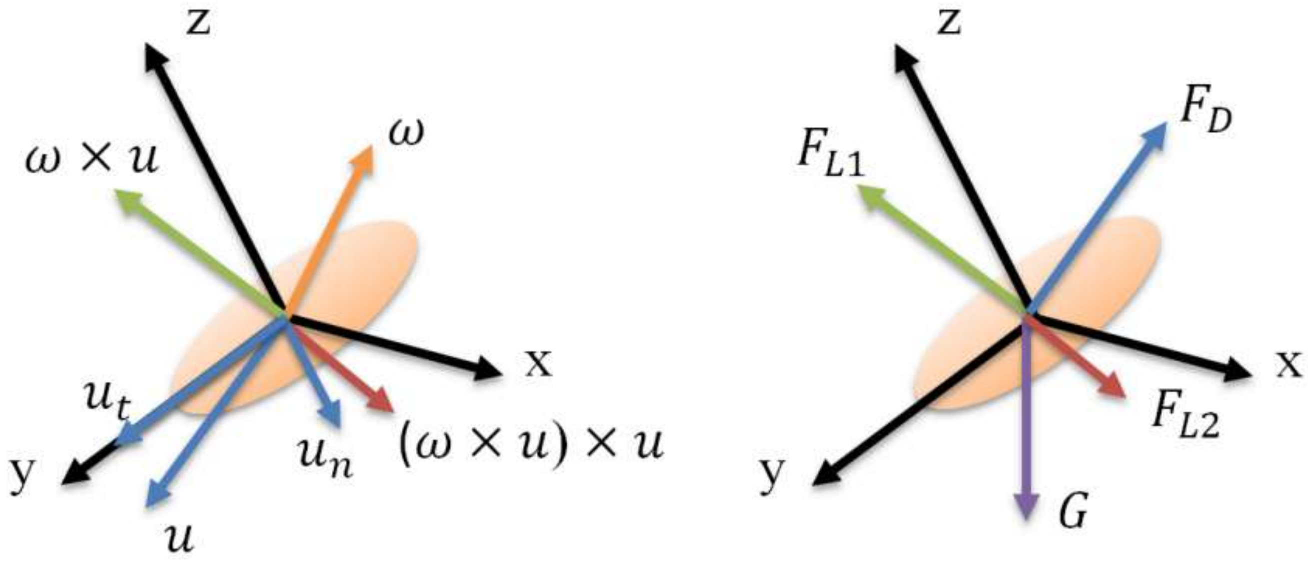
F2. Viscous Effect

Viscous Forces

$$\begin{pmatrix} F_D \\ F_{L1} \\ F_{L2} \end{pmatrix} = \frac{1}{2}\rho_f \|u\|^2 A \begin{pmatrix} -C_d e_1 \\ C_{l1} e_2 \\ C_{l2} e_3 \end{pmatrix}$$

Drag
Rotational Lift
Translation Lift

$$(e_1, e_2, e_3) = (\vec{u}, \vec{\omega} \times \vec{u}, (\vec{\omega} \times \vec{u}) \times \vec{u})$$



Force Coefficients

- Instantaneous coefficients

$$C = C(\alpha, Re)$$

(angle of attack, Reynolds number)

Generalized Parameter Model

$$(C_d, C_{l1}, C_{l2}) = (C_D \sin^2 \alpha, C_{L1} \sin(2\alpha), C_{L2} \cos(2\alpha))$$

$$\alpha \ = \ \tan^{-1}(\|u_n\|/\|u_t\|)$$

Generalized Kirchhoff Equation

Dynamical Model

$$
\begin{aligned}
\mathbf{M}_a \dot{\boldsymbol{u}} &= (\mathbf{M}_a \boldsymbol{u}) \times \boldsymbol{\omega} + F_v \\
\mathbf{I}_a \dot{\boldsymbol{\omega}} &= (\mathbf{I}_a \boldsymbol{\omega}) \times \boldsymbol{\omega} + (\mathbf{M} \boldsymbol{u}) \times \boldsymbol{u} + \Gamma_M
\end{aligned}
$$

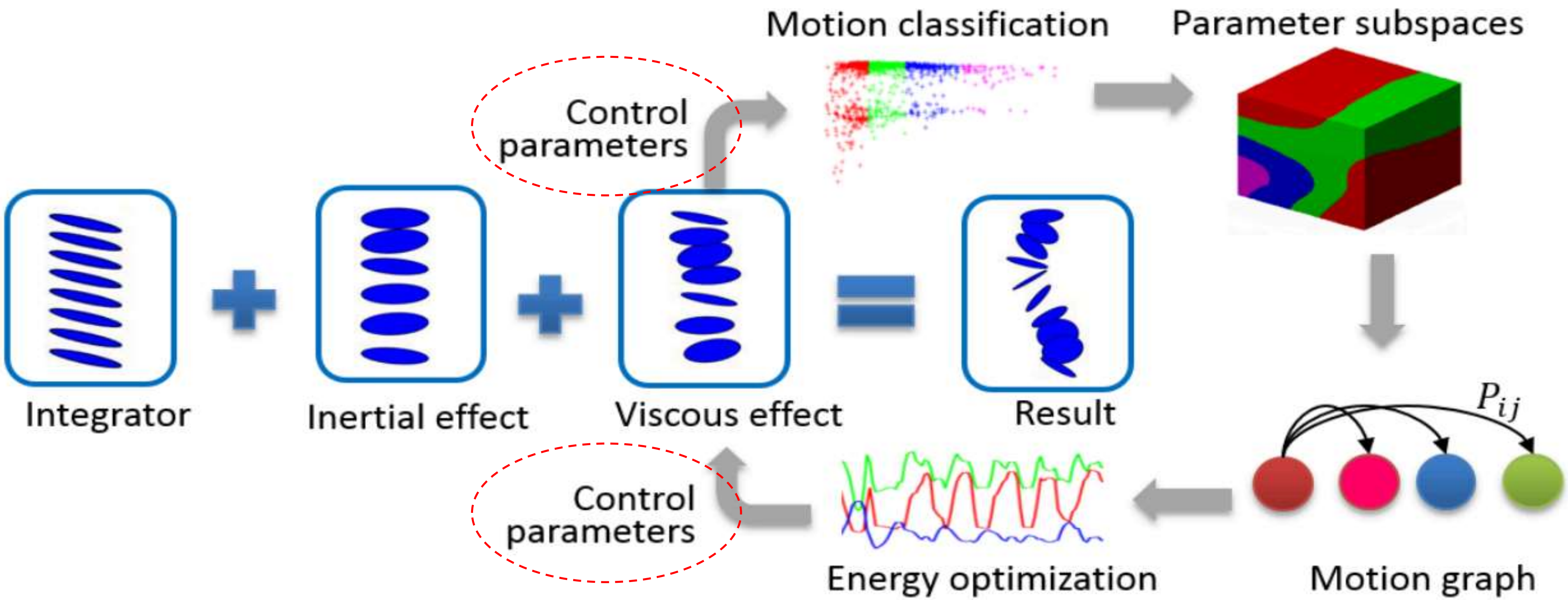
Force decomposition:

$$
\begin{aligned}
F_v(X) &= \begin{pmatrix} F_D + F_{L1} + F_{L2} + F_G \\ \Gamma_M + \Gamma_G \end{pmatrix} \\
&= \begin{pmatrix} F_D + F_{L1} + F_{L2} \\ \Gamma_M \end{pmatrix}(X) + m_f \begin{pmatrix} (\bar{\rho}/\rho_f - 1) R^T g \\ \vec{r} \times R^T g \end{pmatrix}
\end{aligned}
$$

gravity

$$
\Gamma_M = \vec{p} \times (F_D + F_{L1} + F_{L2})
$$

$$
\|\vec{p}\| = (1 - \sin^3 \alpha) a/4
$$



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