Computation Time Reduction Techniques for Model Predictive Control of Hybrid Systems

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Outline of This Presentation

1. Hybrid Systems
2. Model Predictive Control
3. Technical Difficulty and Our Approach
4. Proposed Method
5. Further Extension

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What are Hybrid Systems?

Hybrid Systems: Dynamical Systems with Continuous Variables and Discrete Variables

【Example 1】 Automobile

Accelerator pedal (Torque)

Continuous

Gear 1/Gear 2/Gear 3･･･

Discrete

- Biped walking robot
- Aircraft autopilot modes
- Air and ground transportation
- Chemical plants

…

What are Hybrid Systems?

【Example 1】 Automobile

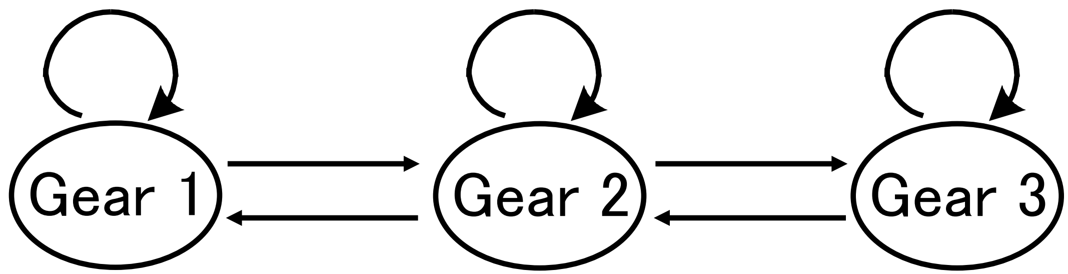
Accelerator pedal

Continuous

Gear 1/Gear 2/Gear 3･･･

Discrete

Finite Automaton:



Dynamics of Automobile in Gear *i* :

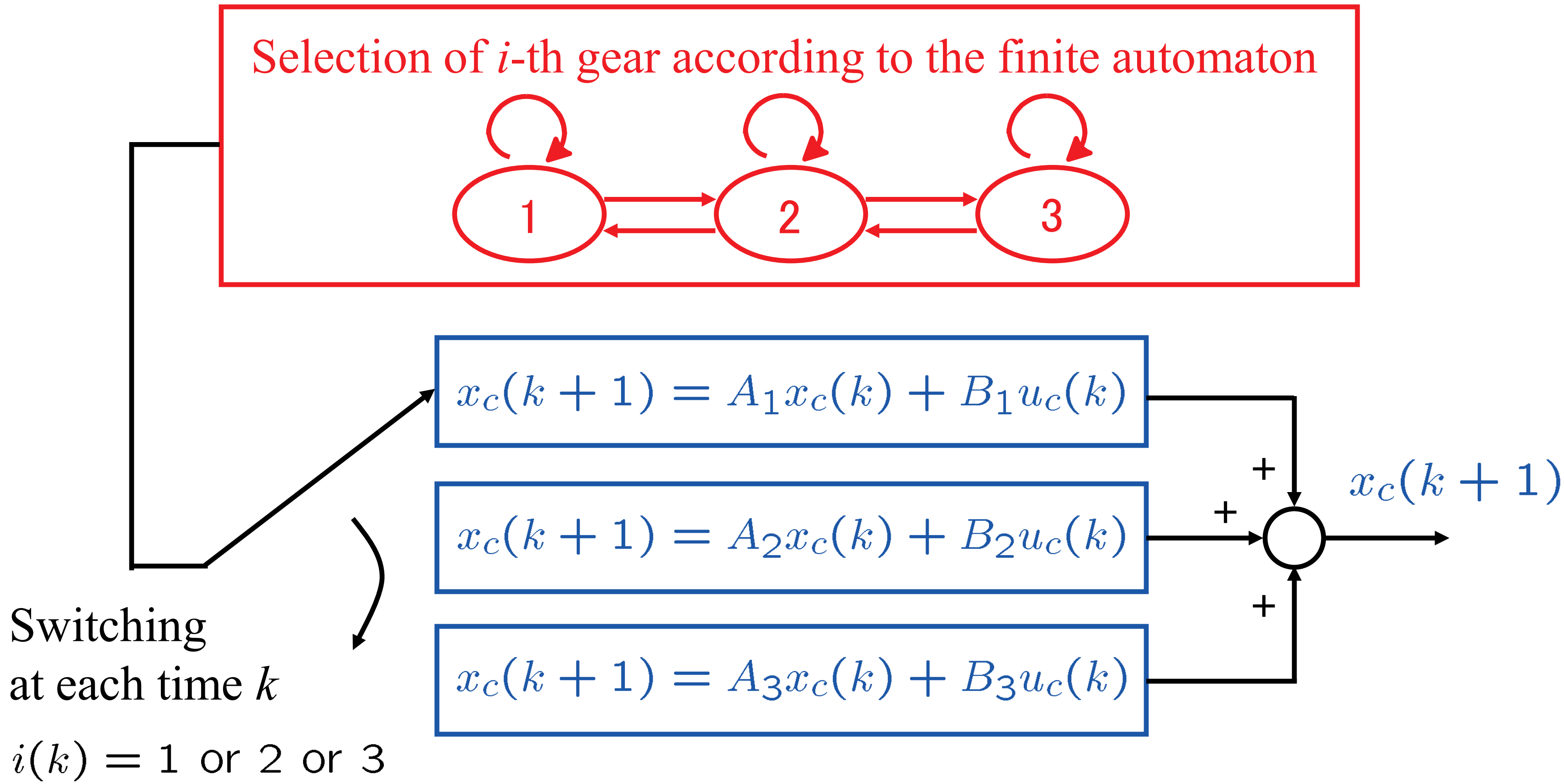
(Discrete-time state equation)

$$x_c(k+1) = A_{i(k)} x_c(k) + B_{i(k)} u_c(k)$$

$$x_c(k) = \begin{bmatrix} \text{Position} \\ \text{Velocity} \end{bmatrix} \qquad u_c(k) = \text{Torque}$$

$$i(k) = 1 \text{ or } 2 \text{ or } 3 \qquad k = 0,\ 1,\ 2,\ \cdots \ \text{Time}$$

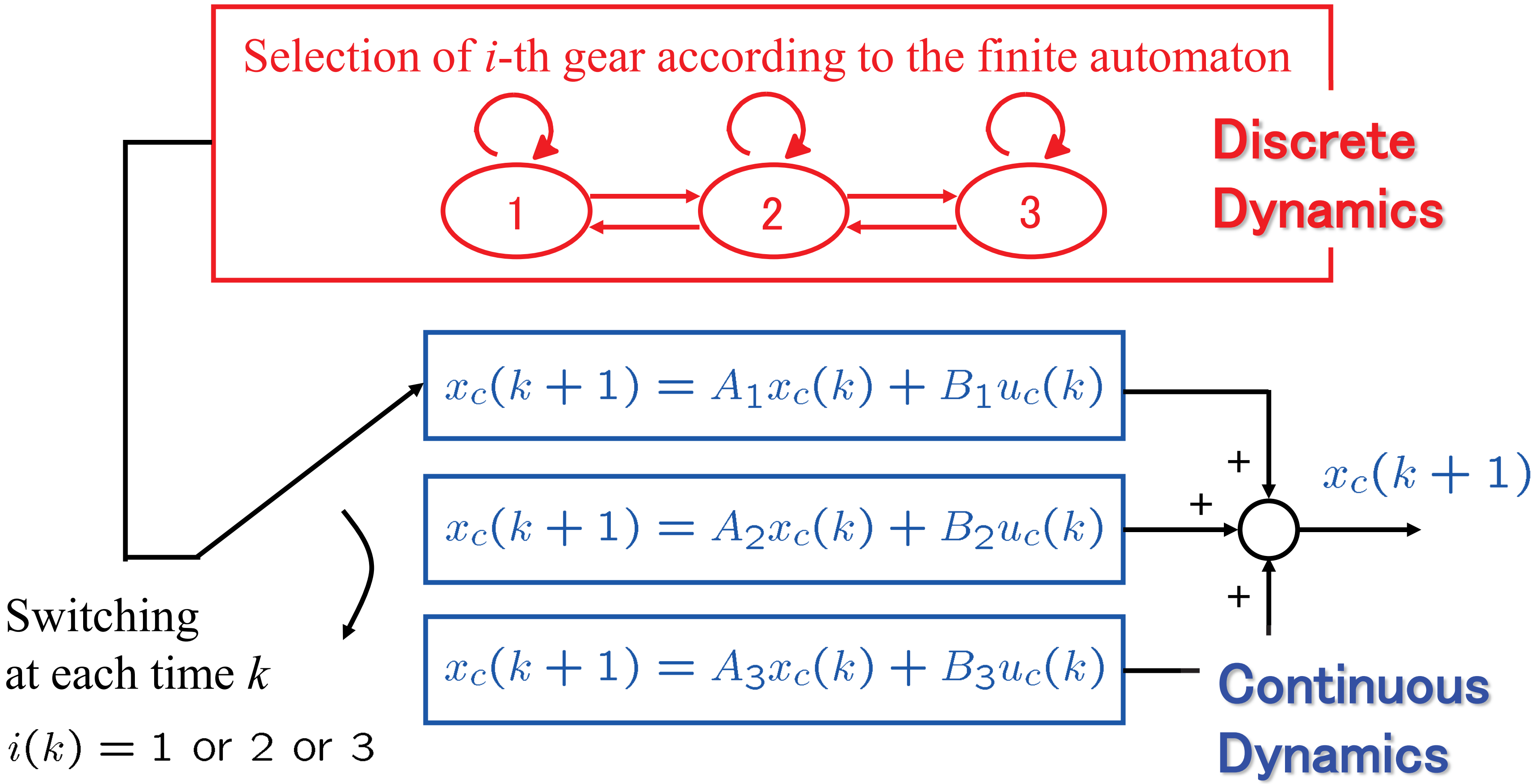
What are Hybrid Systems?



By determining $x_c(k),\ u_c(k),\ i(k)$ at time k ,

$x_c(k+1)$ at next time $k+1$ is uniquely determined.

What are Hybrid Systems?



<u>In general,</u>

(1) Gear 1, Gear 2, Gear 3 $\rightarrow$ mode 1, mode 2, mode 3

(2) Mode transition constraints depend on $x_c(k)$, $u_c(k)$.

(Piecewise Affine Systems)

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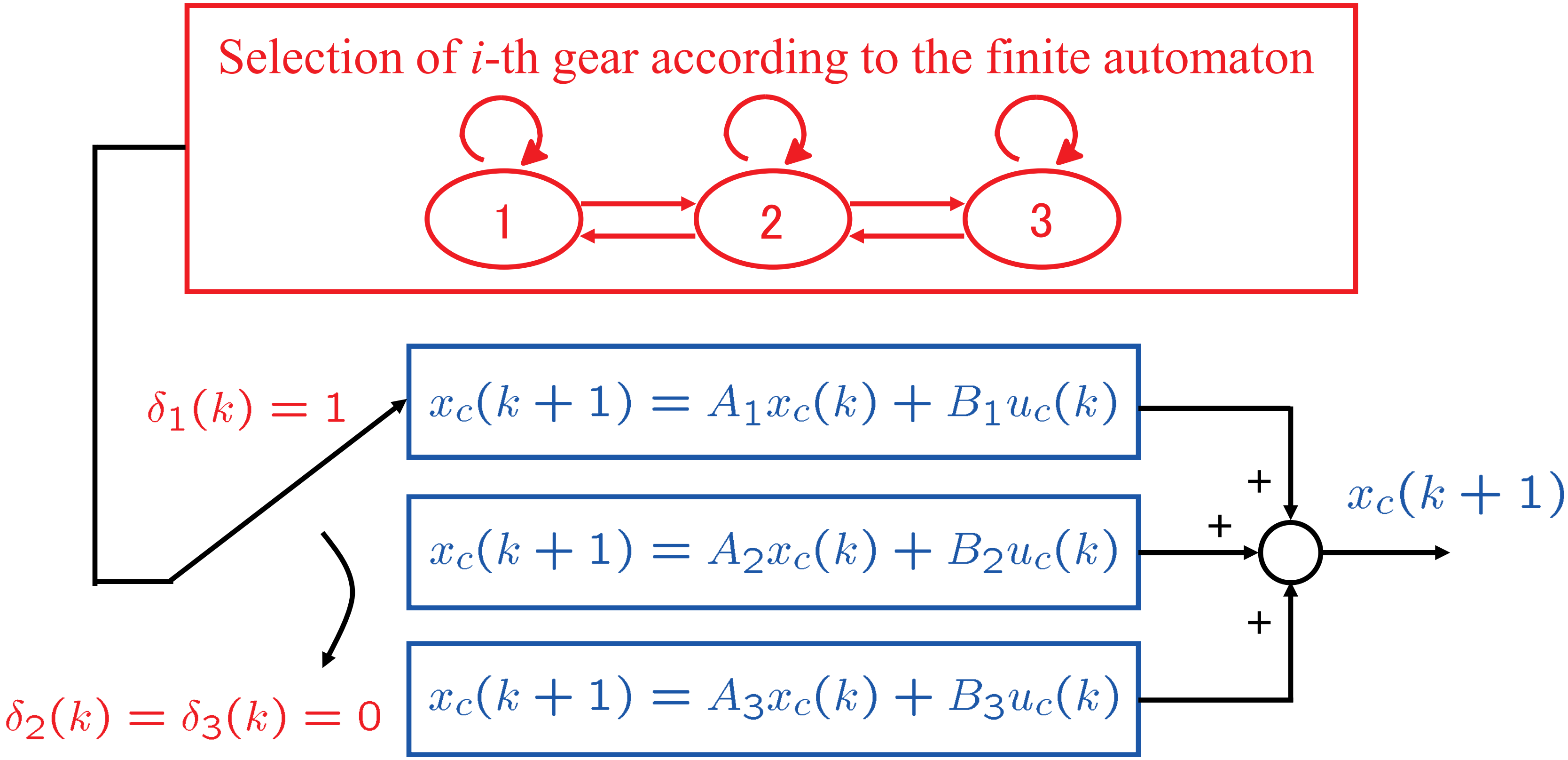
Modeling of Hybrid Systems



Binary variables $\delta_1,\ \delta_2,\ \delta_3$ are used.

$$\begin{cases} \delta_i(k) = 1 & \text{if Gear } i \\ \delta_i(k) = 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \quad x_c(k+1) = \sum_{i=1}^{3} \delta_i(k)\Big(A_i x_c(k) + B_i u_c(k)\Big)$$

+ Model of the finite automaton

Modeling of Hybrid Systems

Model of Hybrid System:

$$x_c(k+1) = \sum_{i=1}^{3} \delta_i(k)\Big(A_i x_c(k) + B_i u_c(k)\Big)$$

+ Model of the finite automaton

$$\Downarrow$$

Mixed Logical Dynamical (MLD) Model:

[Bemporad and Morari 1999]

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ Cx(k) + Du(k) \leq E \end{cases} \qquad \begin{array}{l} x(k) \in \mathbf{R}^{n_c} \times \{0,1\}^{n_d} \\ u(k) \in \mathbf{R}^{m_c} \times \{0,1\}^{m_d} \end{array}$$

$$x(k) = \begin{bmatrix} x_c(k) \\ x_d(k) \end{bmatrix}, \quad x_c(k) \in \mathbf{R}^{n_c}, \quad x_d(k) \in \{0,1\}^{n_d}$$

$$u(k) = \begin{bmatrix} u_c(k) \\ u_d(k) \end{bmatrix}, \quad u_c(k) \in \mathbf{R}^{m_c}, \quad u_d(k) \in \{0,1\}^{m_d}$$

Continuous Dynamics **Discrete Dynamics**

Finite-time Optimal Control Problem

$$\begin{aligned}
\text{find} \quad & u(k) \in \mathbf{R}^{m_c} \times \{0,1\}^{m_d}, \quad k = T,\ T+1,\ \cdots, T+N-1 \\
\text{min} \quad & J = \sum_{i=T}^{T+N-1} \left\{ x'(i)Qx(i) + u'(i)Ru(i) \right\} + x'(N)Q_f x(N) \\
\text{subject to} \quad & \begin{cases} x(k+1) = Ax(k) + Bu(k) \\ Cx(k) + Du(k) \le E \end{cases}
\end{aligned}$$

T : Current time

$\Downarrow$

MIQP Problem (Mixed Integer Quadratic Programming Problem)

$$\begin{aligned}
\text{find} \quad & \bar{u} = \left[u^T(T) \quad u^T(T+1) \quad \cdots \quad u^T(T+N-1) \right]^T \\
\text{min} \quad & \bar{u}^T M_1 \bar{u} + \bar{u}^T M_2 x(T) \\
\text{subject to} \quad & L_1 \bar{u} \le L_2 x(T) + L_3
\end{aligned}$$

The dim. of continuous variables: $m_c N$

The dim. of binary variables: $m_d N$

The computation time is too long for practical plants.

Model Predictive Control

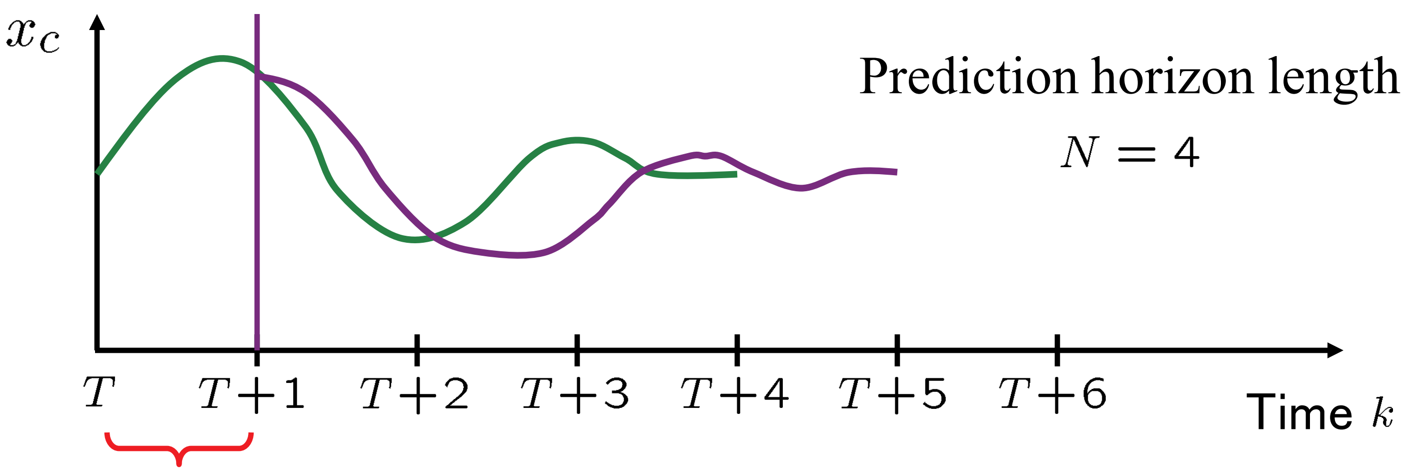
Model Predictive Control …

The finite-time optimal control problem is solved at each time.

Step 1: Derive $\bar{u} = \left[u'(T) \;\; u'(T+1) \;\; \cdots \;\; u'(T+N-1) \right]'$ by solving the MIQP problem.

Step 2: Apply only $u(T)$ to the plant.

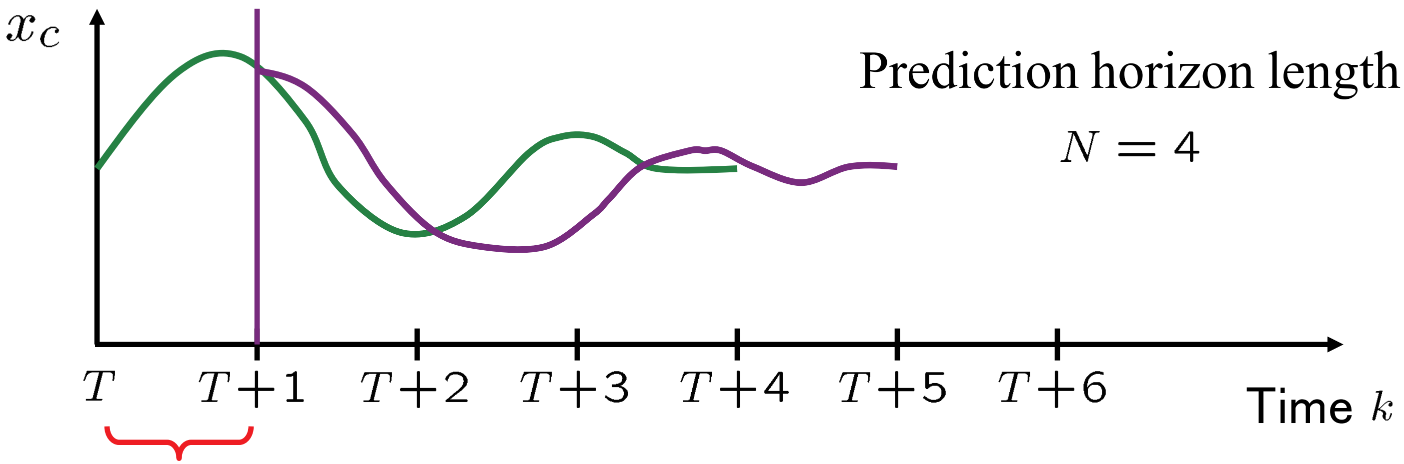
Step 3: Update $T \to T+1$, and go to Step 1.



The MIQP problem must be solved within the sampling period.

'On-line Optimization'

Model Predictive Control



The MIQP problem must be solved within the sampling period.

'On-line Optimization'

Another Approach… **Off-line Optimization**

The multi-parametric MIQP problem is solved.
Then the explicit controller $u^*(k) = f(x(k))$ is derived.

Practical applications are restricted.
Small size systems, Slow dynamical systems, …

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Summary of Our Approach

Technical Difficulty:

The computation time to solve the MIQP problem is too long.

$\Downarrow$ To overcome this difficulty

(1) Improvement of the MIQP solver

(2) Improvement of the modeling of discrete dynamics

→ The computation time depends on the modeling method.

However, few results from the modeling viewpoint

Summary of Our Approach

(2) Improvement of the modeling of discrete dynamics

→ The computation time depends on the modeling method.

The Standard Method: Inequality based modeling

[Bemporad and Morari 1999]

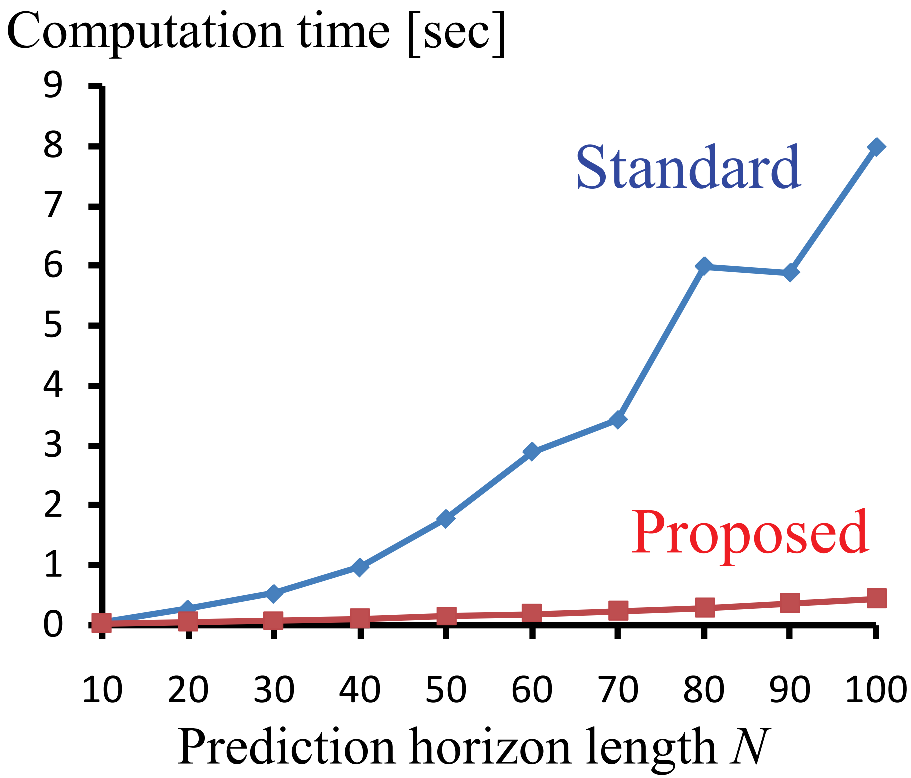
The Proposed Method:

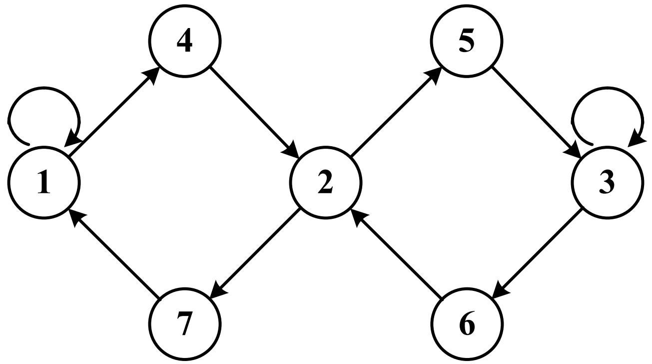
- State-equation based modeling [Kobayashi, Imura 2006, 2007]
- Time-sequence based modeling [Kobayashi, Imura 2007]
- Graph-switching based modeling

Effectiveness of The Proposed Method

Computation time of the finite-time optimal control problem:

- Plant: 2^{nd}-order system with 7-mode switching





CPU: Intel Core2 Duo 3.0GHz
Memory: 2GB
MIQP solver: CPLEX 11.0

Reduction of the computational time is achieved only by changing from the standard to the proposed representation.

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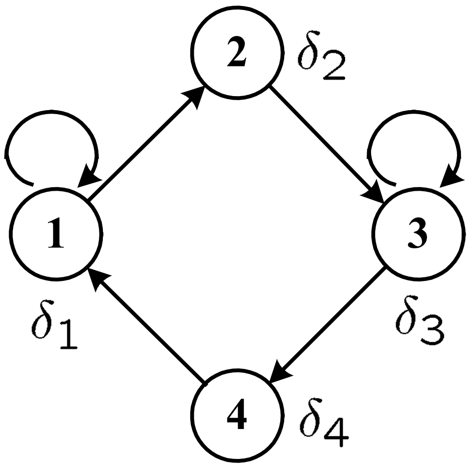
The Standard Method: Inequality Based Modeling



$\delta_i \in \{0,1\},$

$\delta_1+\delta_2+\delta_3+\delta_4=1$

$$\begin{cases} \delta_i(k)=1 & \text{if mode } i \\ \delta_i(k)=0 & \text{otherwise} \end{cases}$$

Propositional logic:

$$[\delta_1(k)=1] \rightarrow [\delta_1(k+1)=1] \vee [\delta_2(k+1)=1] \qquad \text{(mode 1)}$$

$$\vdots$$

$$\Downarrow$$

Linear inequalities: $\underbrace{\delta_1(k)}_{x_{d1}(k)} \leq \underbrace{\delta_1(k+1)}_{u_{d1}(k)} + \underbrace{\delta_2(k+1)}_{u_{d2}(k)}$

$$\vdots$$

$$\Downarrow$$

A combination of state equation and inequality:

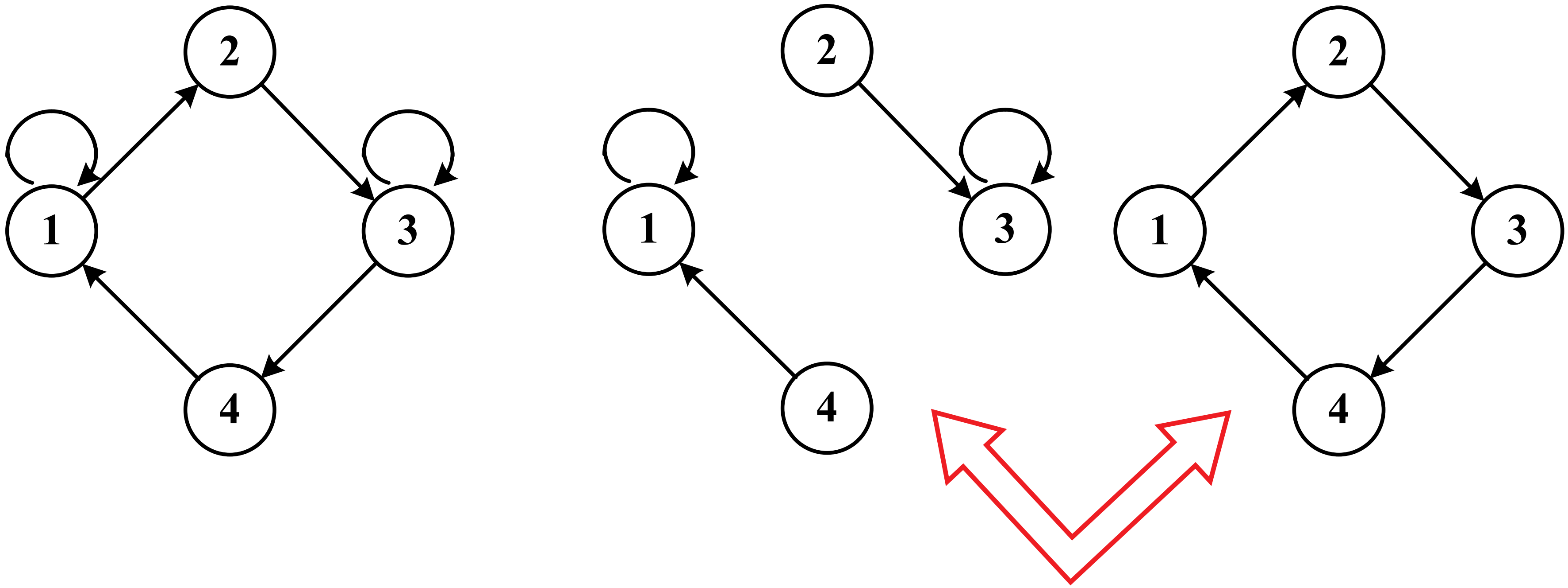
$$\begin{cases} x_d(k+1)=u_d(k) \\ C_d x_d(k)+D_d u_d(k) \leq E_d \end{cases}$$

MLD model

$$\begin{cases} x(k+1)=Ax(k)+Bv(k) & x \in \mathbf{R}^{n_c} \times \{0,1\}^{n_d} \\ Cx(k)+Dv(k) \leq E & v \in \mathbf{R}^{m_c} \times \{0,1\}^{m_d} \end{cases}$$

The Key Idea of The Proposed Method

Finite Automaton ⇨ **Decomposed to two finite automata**

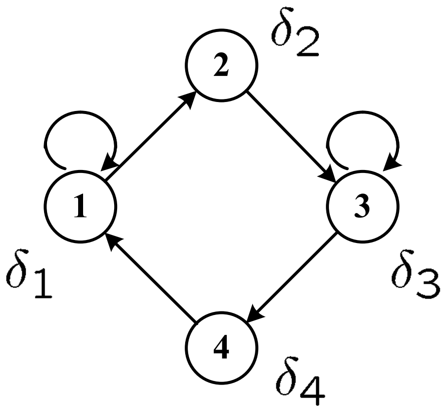


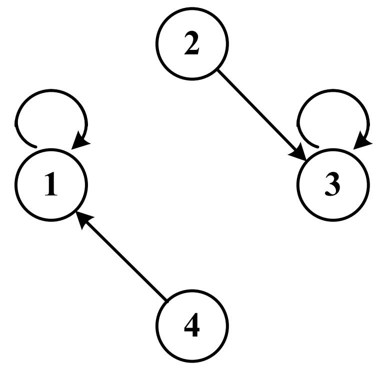
For a given initial mode, by selecting one of finite automata, the mode at the next time is uniquely determined.

⇨ The branch and bound algorithm works effectively.

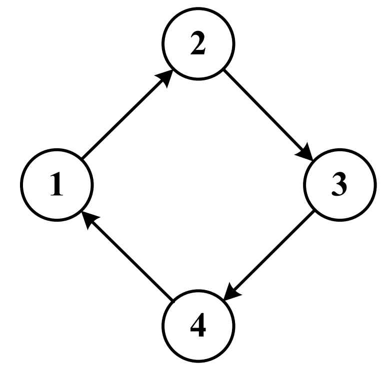
Procedure For Deriving The Proposed Model

Finite Automaton $\Longrightarrow$ **Decomposed to two finite automata**





$\delta(k) = 1$



$\delta(k) = 0$

<u>Model based on the adjacency matrix</u>

$$\begin{bmatrix} \delta_1(k+1) \\ \delta_2(k+1) \\ \delta_3(k+1) \\ \delta_4(k+1) \end{bmatrix} = \begin{bmatrix} \delta(k) & 0 & 0 & 1 \\ 1-\delta(k) & 0 & 0 & 0 \\ 0 & 1 & \delta(k) & 0 \\ 0 & 0 & 1-\delta(k) & 0 \end{bmatrix} \begin{bmatrix} \delta_1(k) \\ \delta_2(k) \\ \delta_3(k) \\ \delta_4(k) \end{bmatrix}$$

$$\delta_1(k) + \delta_2(k) + \delta_3(k) + \delta_4(k) = 1$$

Procedure For Deriving The Proposed Model

Model based on the adjacency matrix

$$\begin{bmatrix} \delta_1(k+1) \\ \delta_2(k+1) \\ \delta_3(k+1) \\ \delta_4(k+1) \end{bmatrix} = \begin{bmatrix} \delta(k) & 0 & 0 & 1 \\ 1-\delta(k) & 0 & 0 & 0 \\ 0 & 1 & \delta(k) & 0 \\ 0 & 0 & 1-\delta(k) & 0 \end{bmatrix} \begin{bmatrix} \delta_1(k) \\ \delta_2(k) \\ \delta_3(k) \\ \delta_4(k) \end{bmatrix}$$

$$\delta_1(k) + \delta_2(k) + \delta_3(k) + \delta_4(k) = 1$$

By defining $u_1(k) := \delta(k)\delta_1(k),\ u_2(k) := \delta(k)\delta_3(k)$,
we obtain

$$\begin{bmatrix} \delta_1(k+1) \\ \delta_2(k+1) \\ \delta_3(k+1) \\ \delta_4(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_1(k) \\ \delta_2(k) \\ \delta_3(k) \\ \delta_4(k) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}.$$

Procedure For Deriving The Proposed Model

Nonlinear polynomial terms can be transformed into linear inequalities as follows.

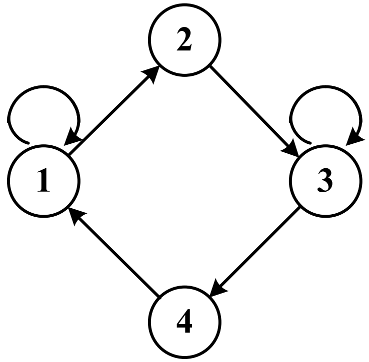
$$u_1(k) = \delta(k)\delta_1(k) \Leftrightarrow \begin{cases} \delta_1(k) - u_1(k) + \delta(k) \leq 1 \\ -\delta_1(k) + 2u_1(k) - \delta(k) \leq 0 \end{cases}$$

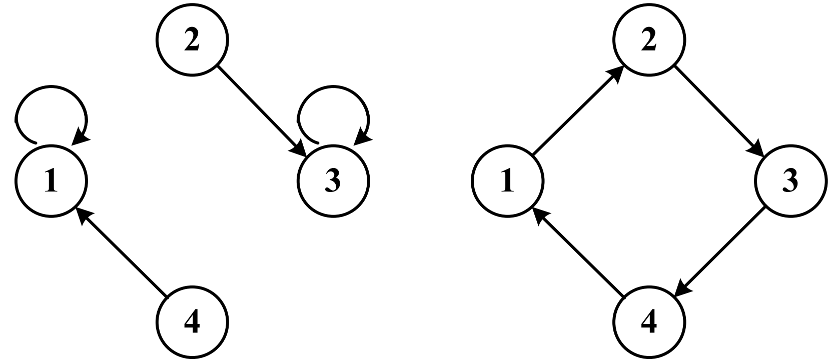
$$u_2(k) = \delta(k)\delta_3(k) \Leftrightarrow \begin{cases} \delta_3(k) - u_2(k) + \delta(k) \leq 1 \\ -\delta_3(k) + 2u_2(k) - \delta(k) \leq 0 \end{cases}$$

[Cavalier *et al.* 1990]

Procedure For Deriving The Proposed Model

Finite Automaton $\Longrightarrow$ **Decomposed to two finite automata**





$$\begin{bmatrix} \delta_1(k+1) \\ \delta_2(k+1) \\ \delta_3(k+1) \\ \delta_4(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_1(k) \\ \delta_2(k) \\ \delta_3(k) \\ \delta_4(k) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}$$

$$\begin{cases} \delta_1(k) - u_1(k) + \delta(k) \leq 1 \\ -\delta_1(k) + 2u_1(k) - \delta(k) \leq 0, \end{cases} \quad \begin{cases} \delta_3(k) - u_2(k) + \delta(k) \leq 1 \\ -\delta_3(k) + 2u_2(k) - \delta(k) \leq 0 \end{cases}$$

$$\delta_1(0), \delta_2(0), \delta_3(0), \delta_4(0) \in \{0,1\}, \quad \sum_{i=1}^{4} \delta_i(0) = 1$$

$$\delta_1(k), \delta_2(k), \delta_3(k), \delta_4(k) \in \mathbf{R}, \; u_1(k), u_2(k), \delta(k) \in \{0,1\}$$

Modeling of Hybrid Systems

Model of Hybrid System:

$$x_c(k+1) = \sum_{i=1}^{3} \delta_i(k)\Big(A_i x_c(k) + B_i u_c(k)\Big)$$

+ Model of the finite automaton

$$\Downarrow$$

Mixed Logical Dynamical (MLD) Model:

[Bemporad and Morari 1999]

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ Cx(k) + Du(k) \leq E \end{cases} \qquad \begin{array}{l} x(k) \in \mathbf{R}^{n_c} \times \{0,1\}^{n_d} \\ u(k) \in \mathbf{R}^{m_c} \times \{0,1\}^{m_d} \end{array}$$

$$x(k) = \begin{bmatrix} x_c(k) \\ x_d(k) \end{bmatrix}, \quad \boxed{x_c(k) \in \mathbf{R}^{n_c},} \quad \boxed{x_d(k) \in \{0,1\}^{n_d}}$$

$$u(k) = \begin{bmatrix} u_c(k) \\ u_d(k) \end{bmatrix}, \quad \boxed{u_c(k) \in \mathbf{R}^{m_c},} \quad \boxed{u_d(k) \in \{0,1\}^{m_d}}$$

Continuous Dynamics **Discrete Dynamics**

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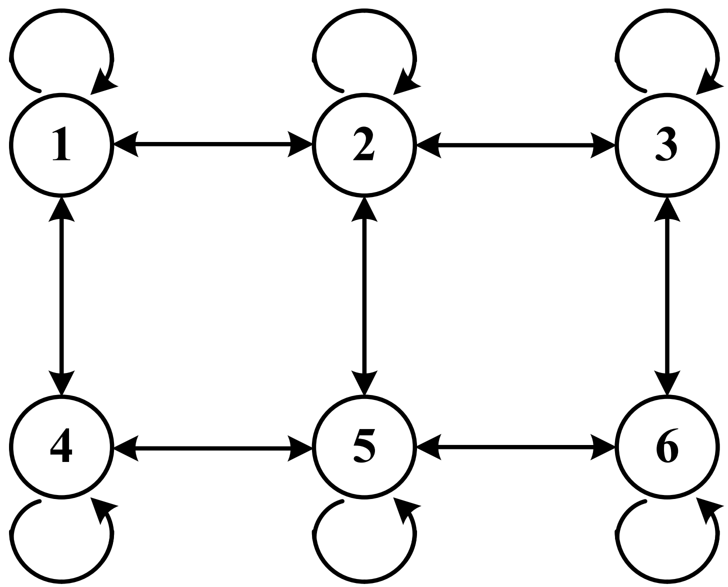
Further Extension

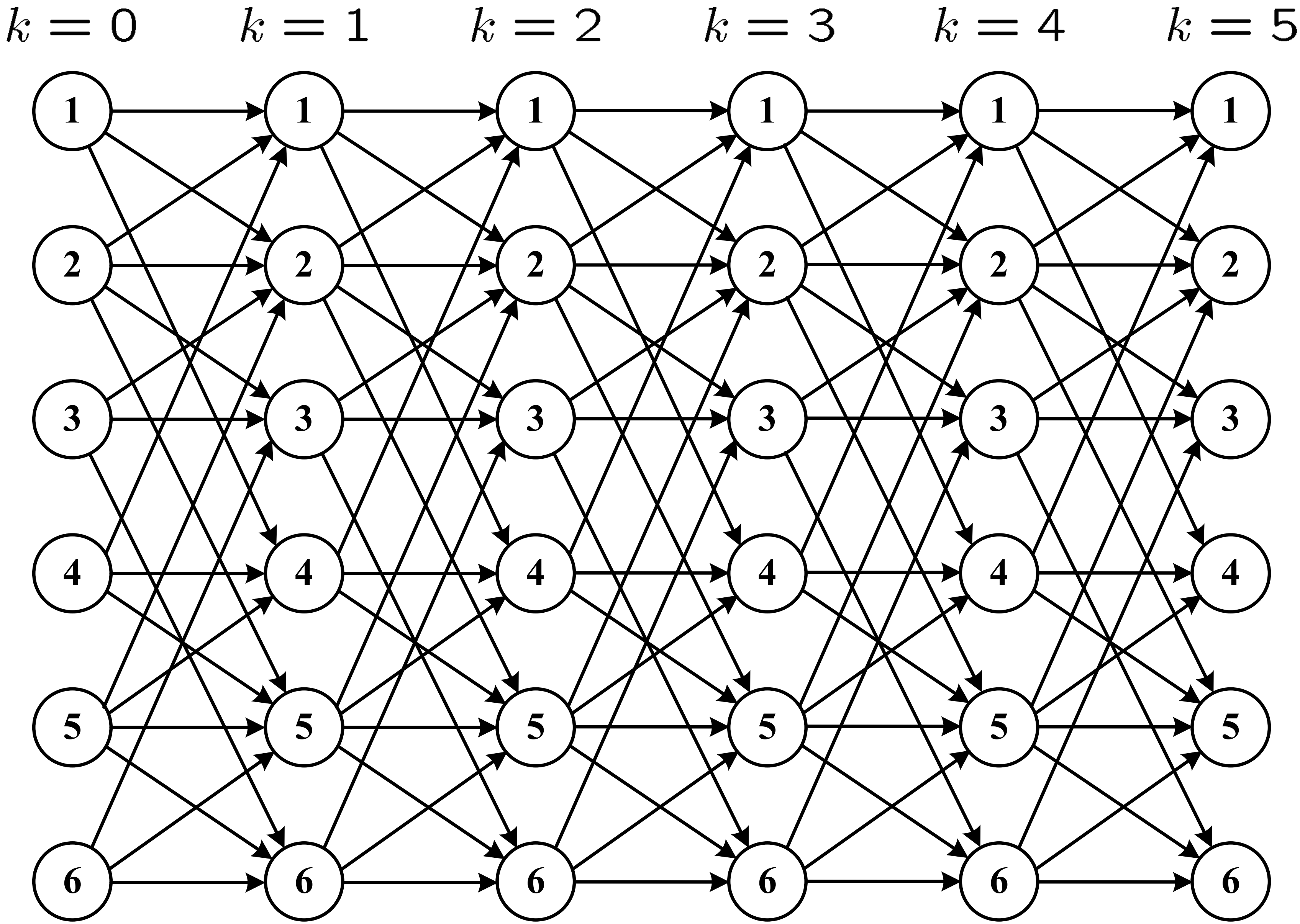
As a further approach for reducing the computation time,

let us approximate a time sequence of finite automata.

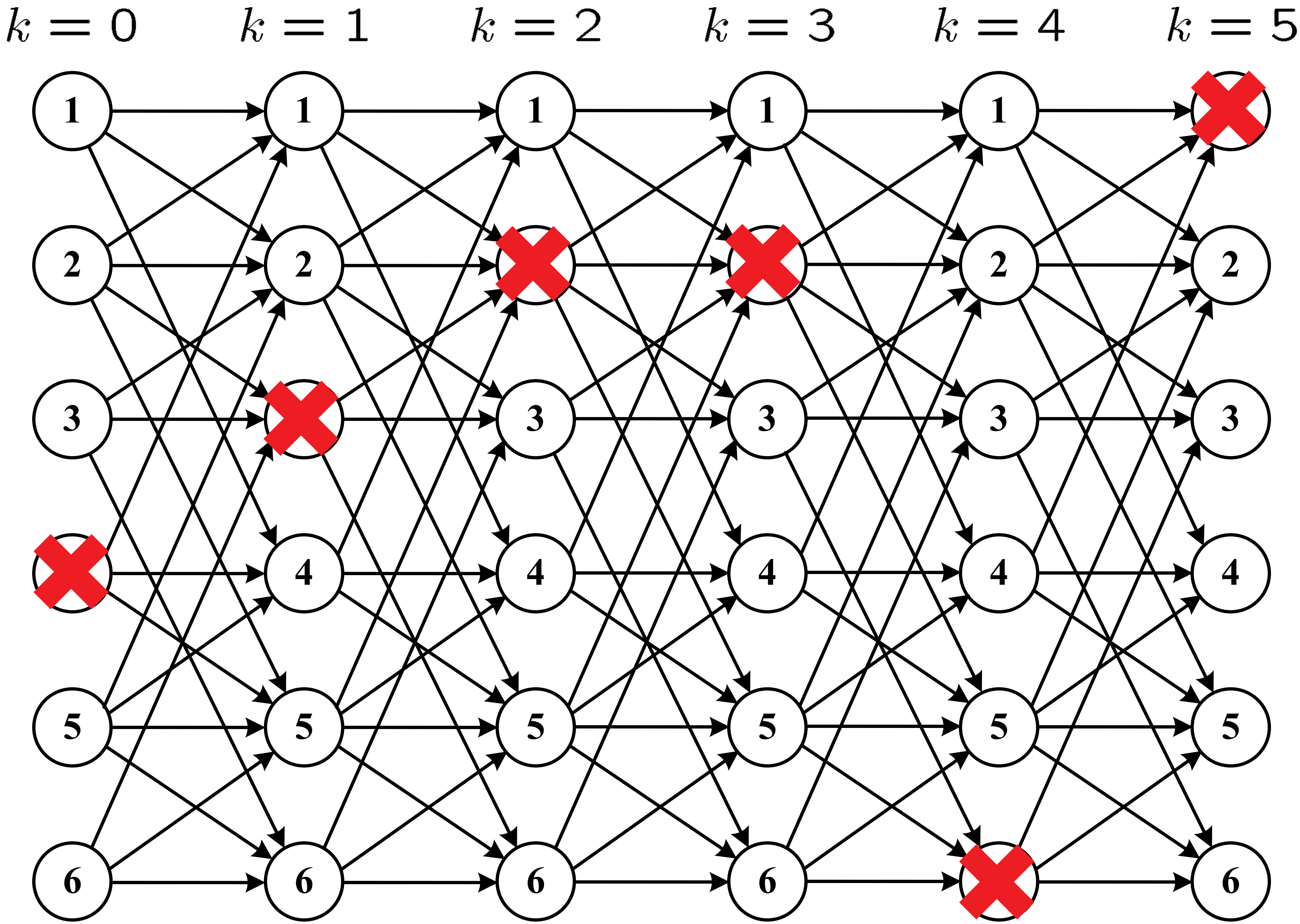
Example:



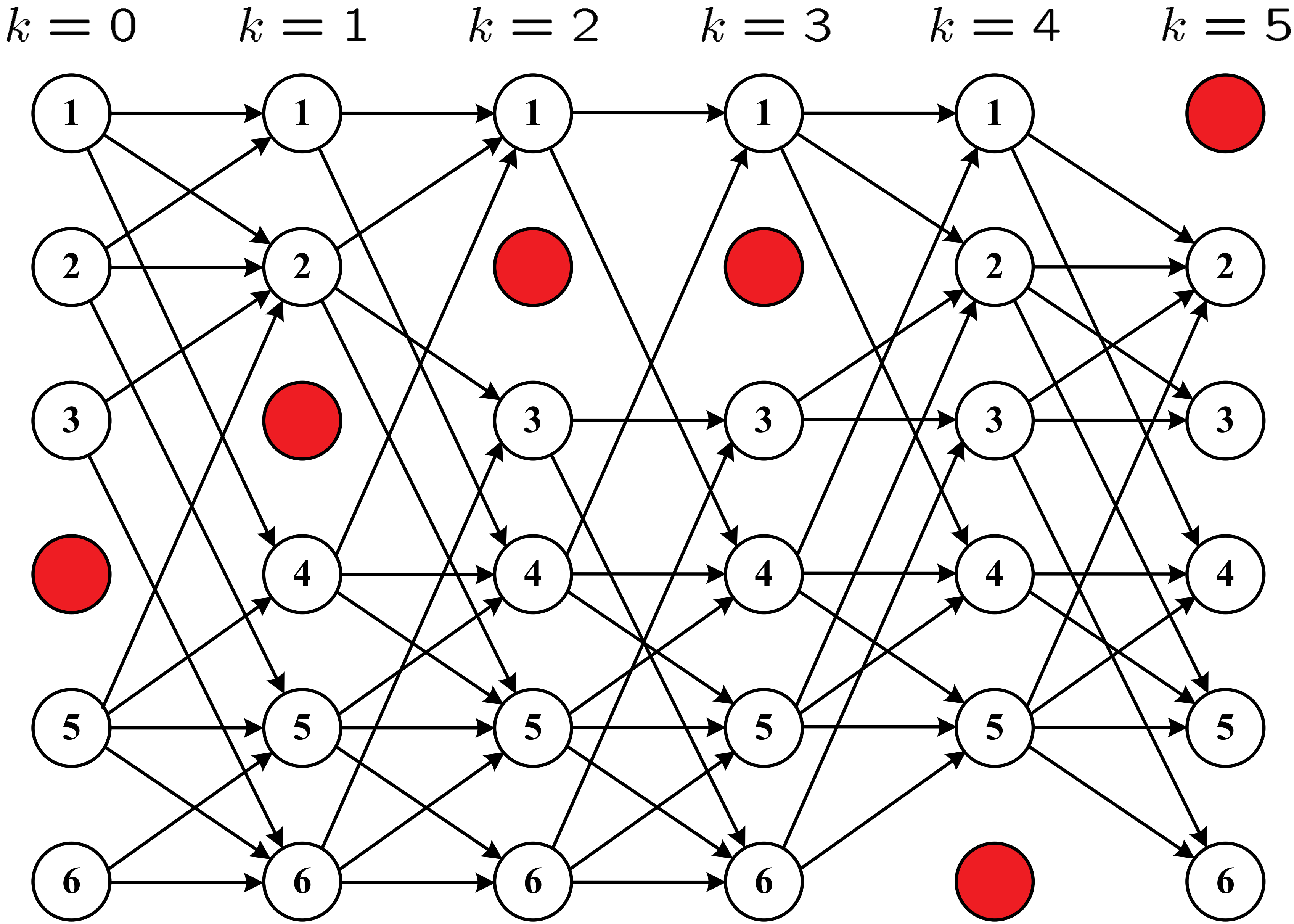
Time Sequence of Finite Automaton



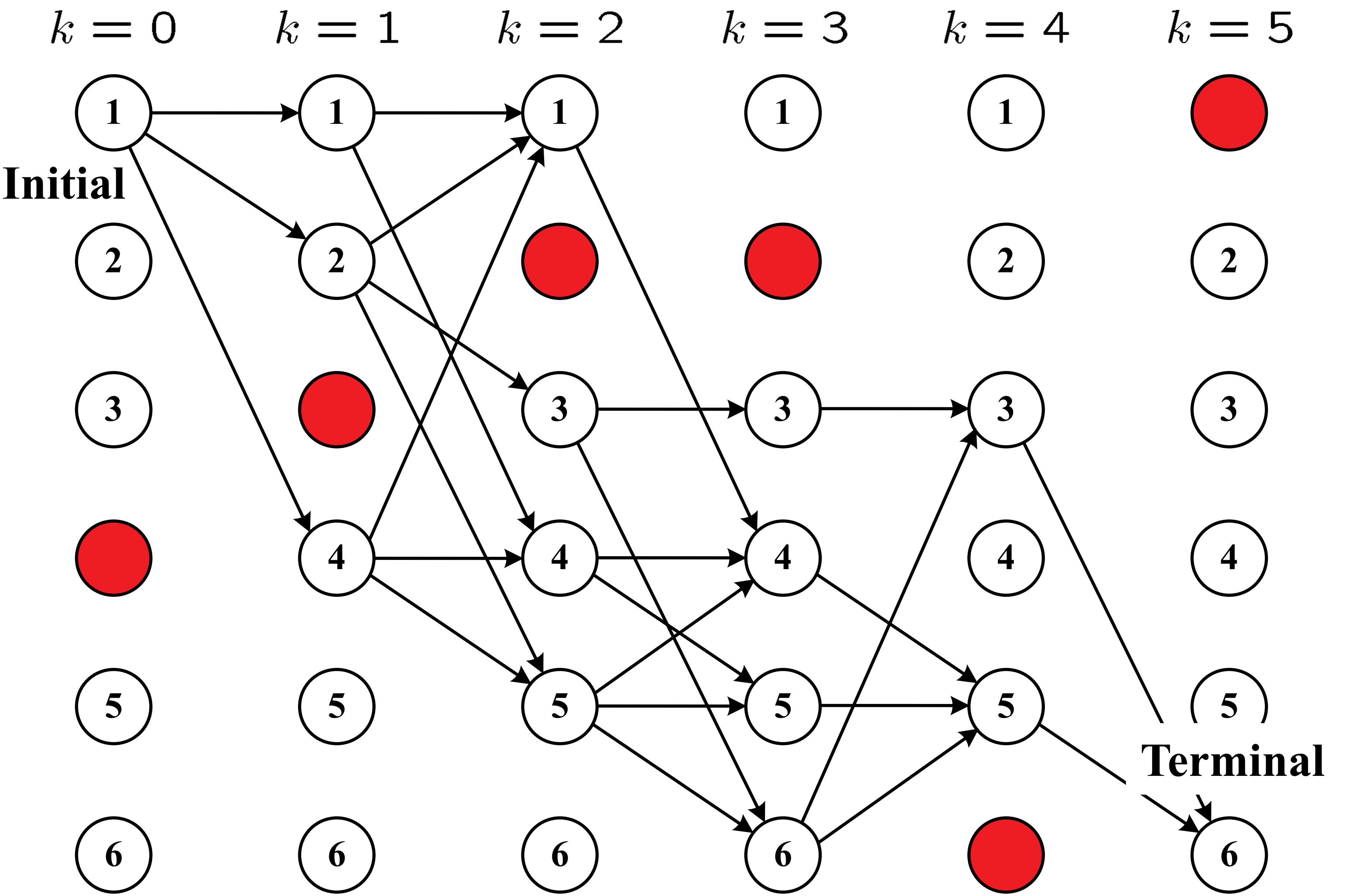
If there are temporal logic constraints,



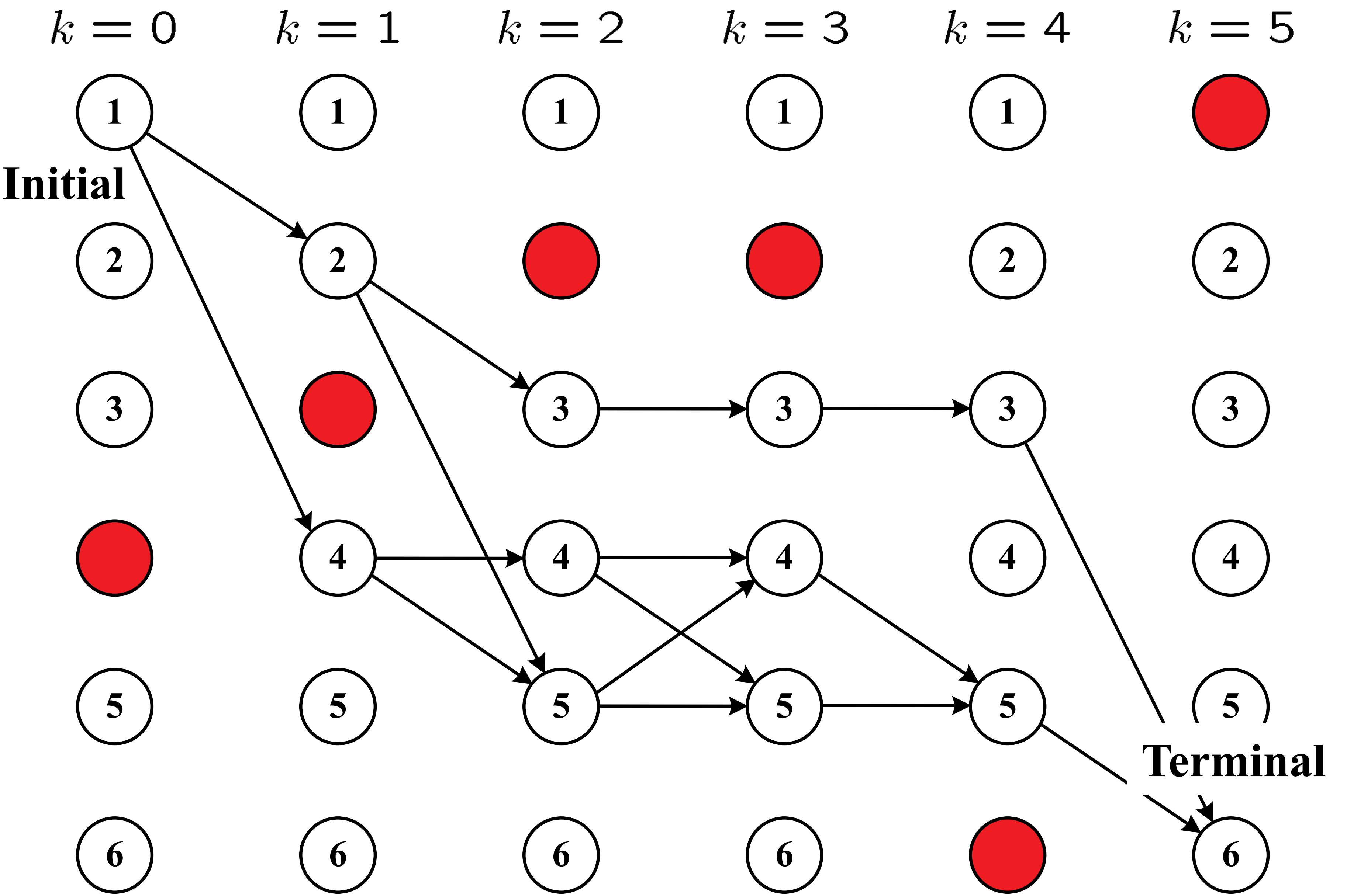
then the time sequence is limited.



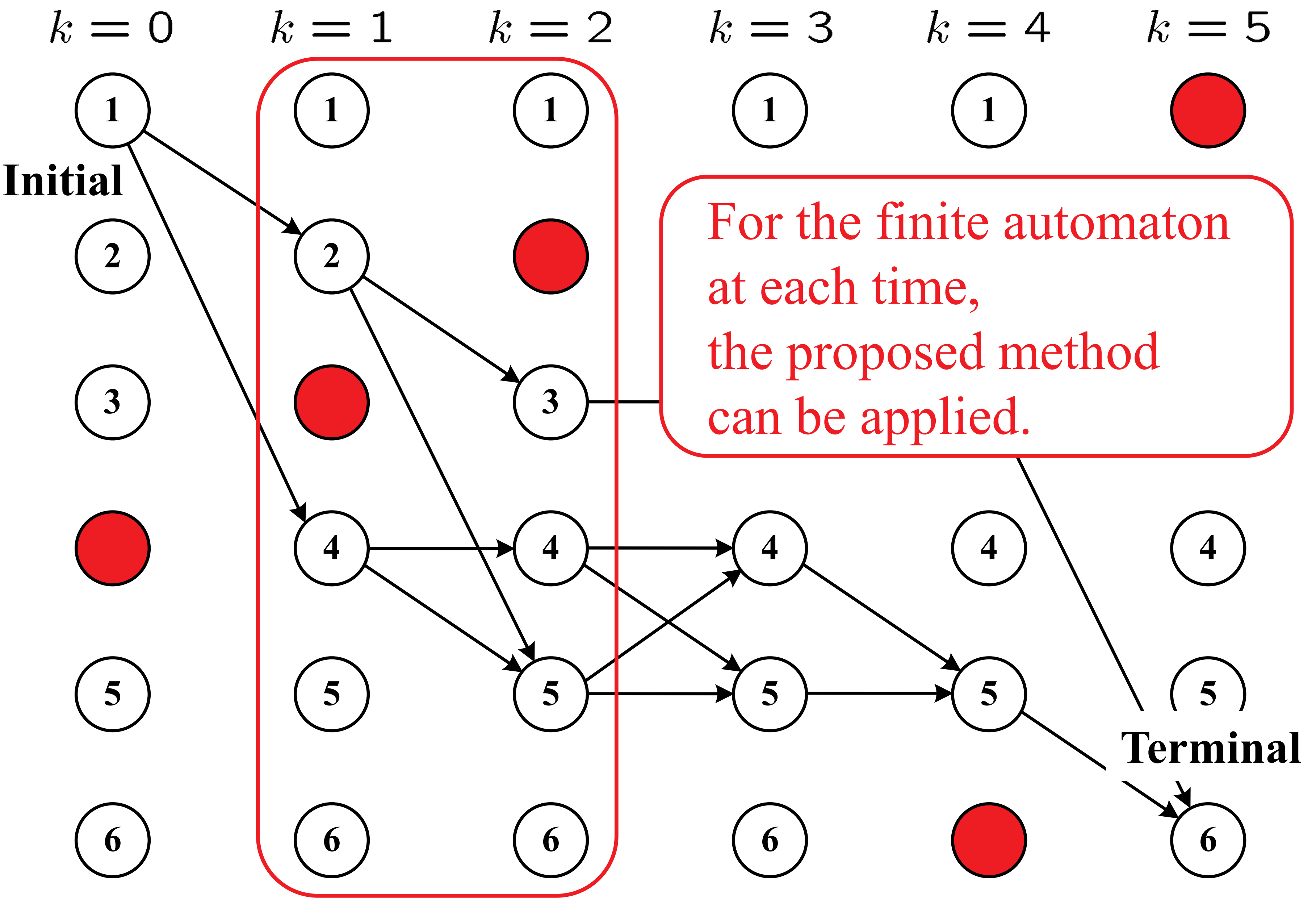
If initial and terminal modes are given, …



Approximate Time Sequence → Suitable??



Approximate Time Sequence → Suitable??



Conclusion

【 Contents of This Presentation 】

（1） Difficulty of hybrid systems control

（2） Introduction to our framework:
Modeling of Finite Automata
for Computation Time Reduction

（3） Further approach: focus on time sequences

【 Future Work 】

（1） Improved algorithm based on
both the on-line and the off-line optimizations