Performance Evaluation of Workflows Using Continuous Petri Nets with Interval Firing Speeds

Kunihiko Hiraishi
School of Information Science,
Japan Advanced Institute of Science and Technology

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ADVANCED INSTITUTE OF
SCIENCE AND TECHNOLOGY
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Motivation

- State space explosion in discrete-state systems
  - Necessity of methods *scalable* for the size of the model.
- Continuous relaxation in optimization problems
  - Necessary condition for the feasibility.
- Fluidification (or continuization):
  Approximating discrete transitions by fluid-flow.
  - Approximation of the reachability set.
  - Approximation of transient behavior of state variables.

Fluidification in Petri Nets

- Formalism
  - Continuous Petri Nets (Autonomous / Timed),
  - Hybrid Petri Nets,
  - Fluid Stochastic Petri Nets, etc.
- Analysis methods
  - Steady-state analysis.
  - Numerical simulations for transient behavior analysis.

Proposed approach

GSPN (Generalized Stochastic Petri Nets)

↓ Fluidification

TCPN (Timed Continuous Petri Nets)

↓ Transformation

PWLS (PieceWise Linear Systems)

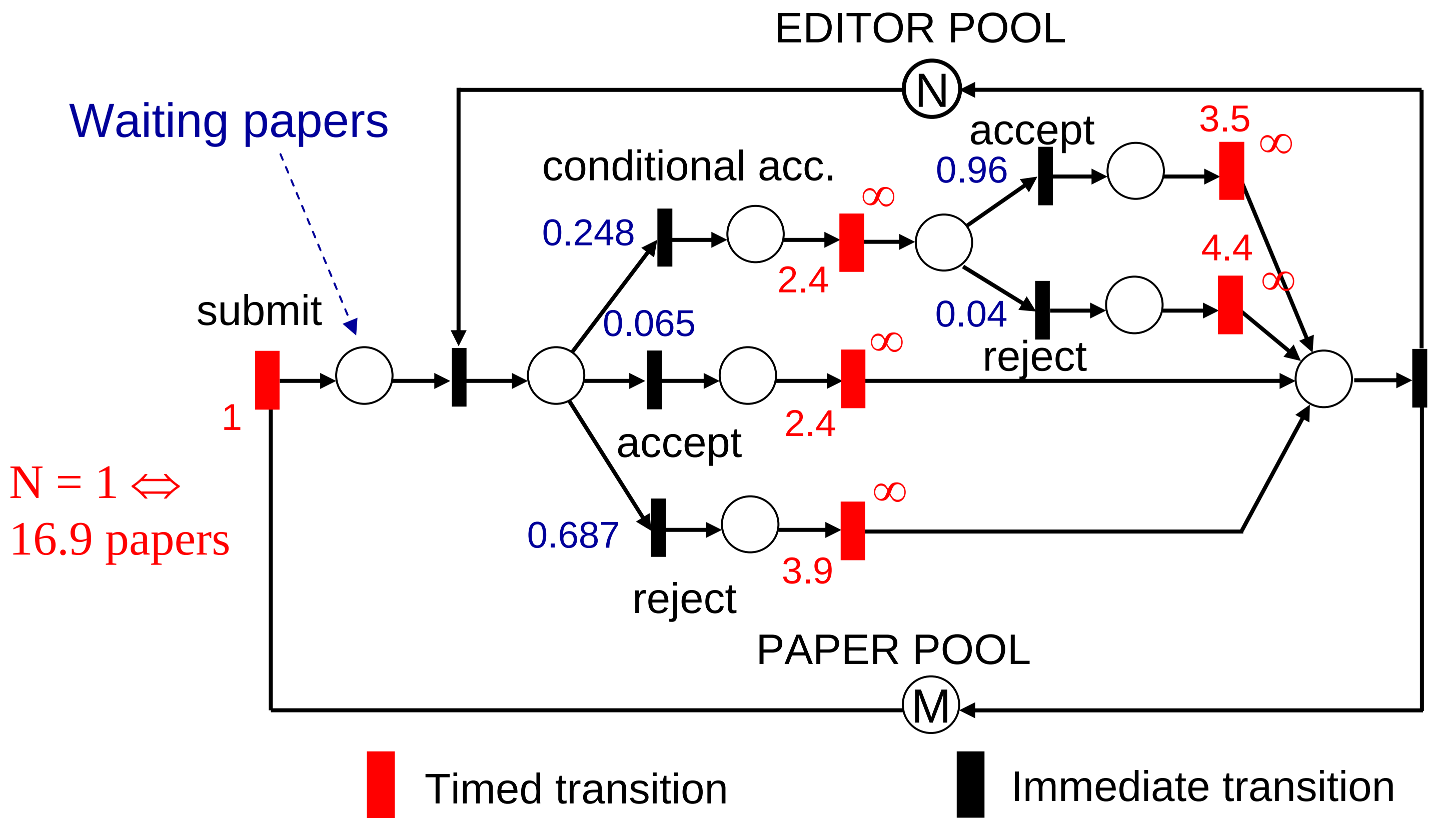
↓ Interval Method

Transient behavior
(Guaranteed enclosures for state vector)

What's new is the approach

- Computation of guaranteed enclosure in transient analysis: all possibilities are computed at the same time.
- Interval firing speeds for approximating probabilistic deviation.
- Using place invariants in piecewise interval computation as constraints.
- A prototyping computer tool.

GSPN model



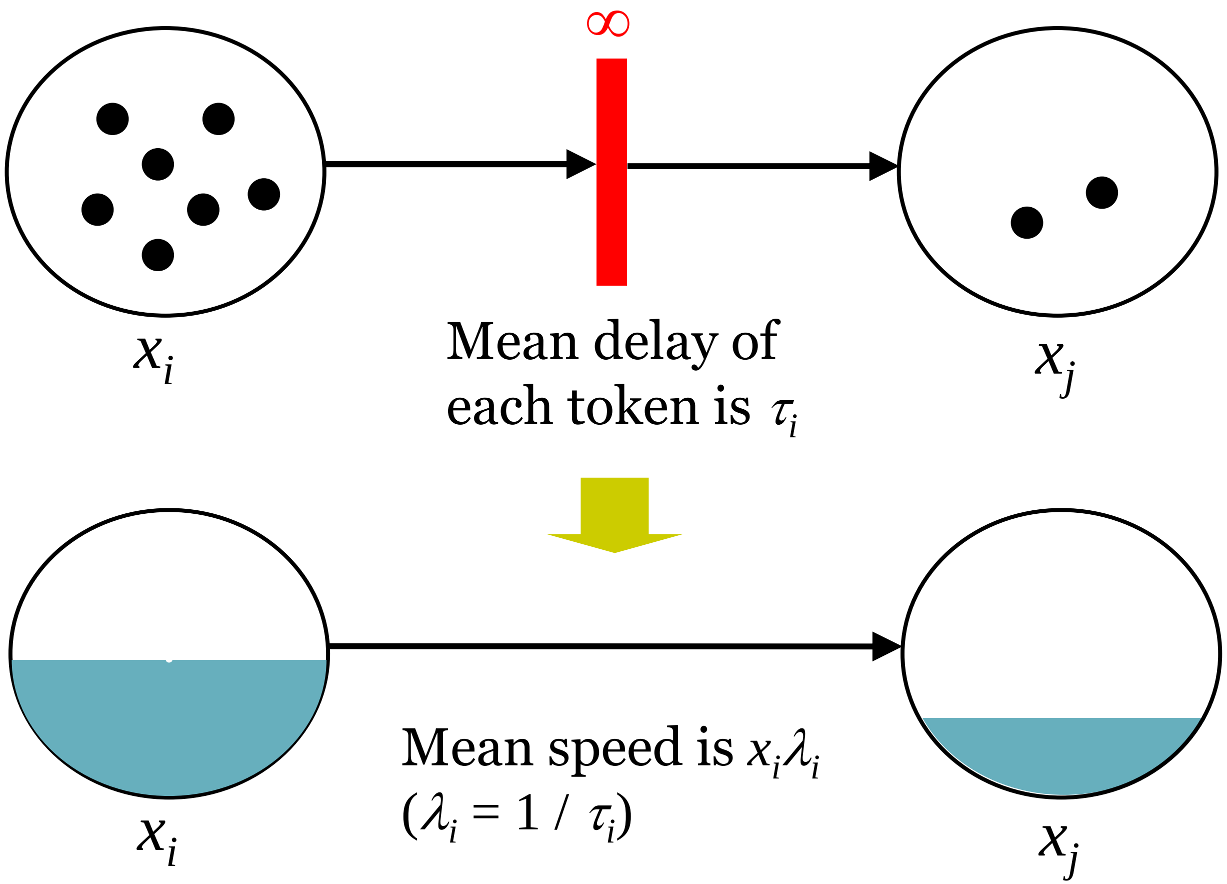
Paper review workflow

Computation results by DSPNExpress-NG

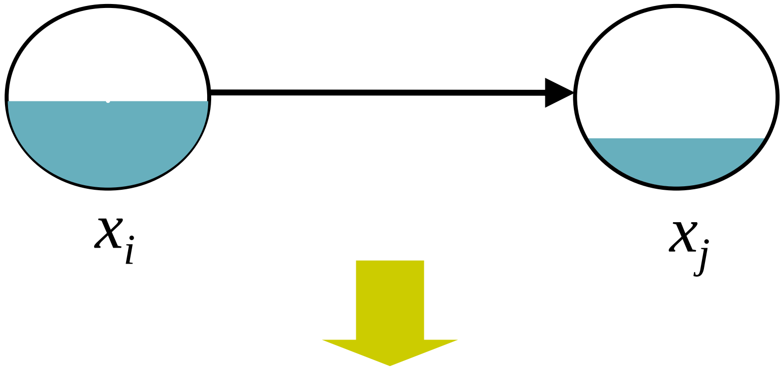
N	#states	CPU Time (sec.)	#Waiting papers	p(#paper pool = 0)
3	2926	0.3	10.18	0.30
4	8866	0.7	5.94	0.094
5	23023	2.3	1.99	0.013
6	53053	6.2	0.63	0.0021
7	110968	15	0.21	0.00049
8	213928	29	0.08	0.00020
9	384098	58	0.03	0.00010

Itanium2 1.6GHz/9MBCache, 16GB Memory

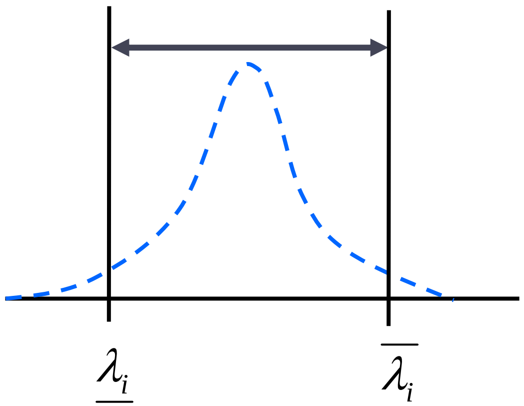
Fluidification



Approximating probabilistic deviation



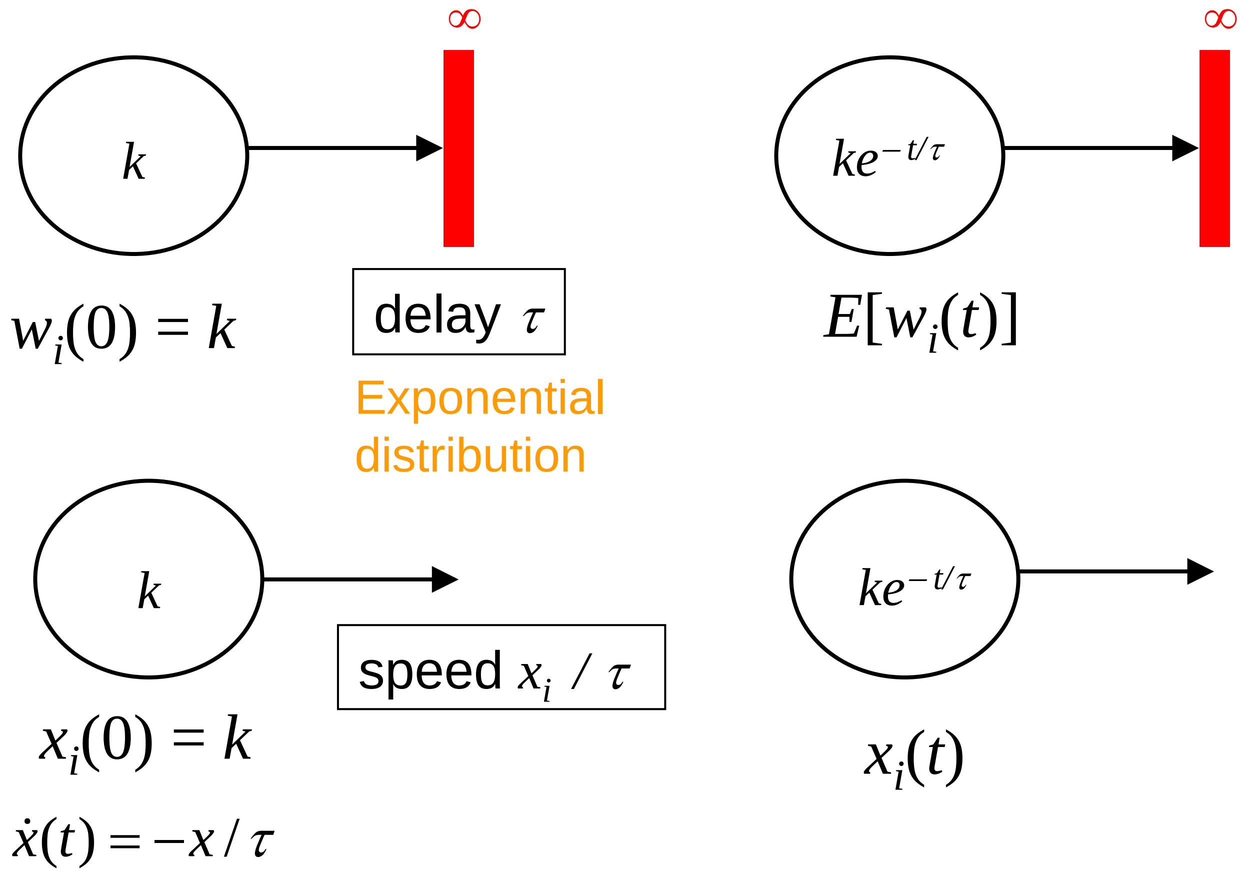
$$\dot{x}_i = -[\lambda_i] \cdot x_i$$



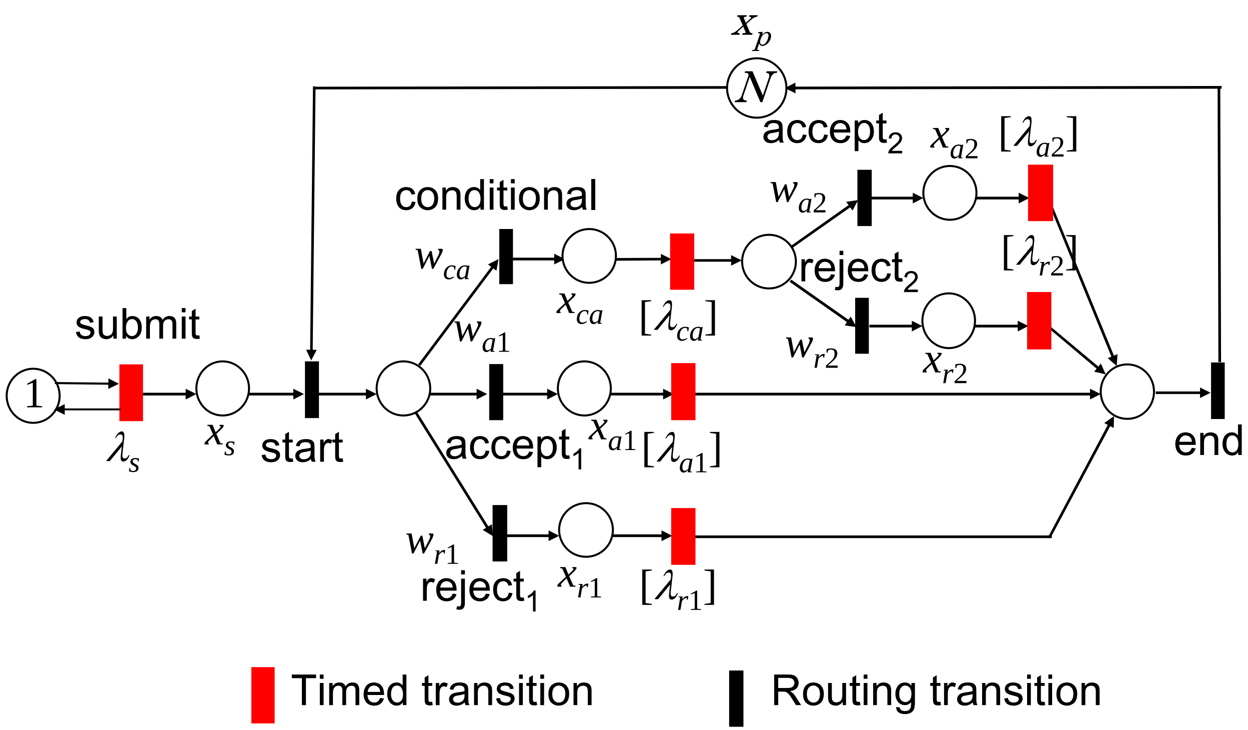
$$[\lambda_i] = [\underline{\lambda_i}, \overline{\lambda_i}]$$

Interval firing speed

Preserving expected values

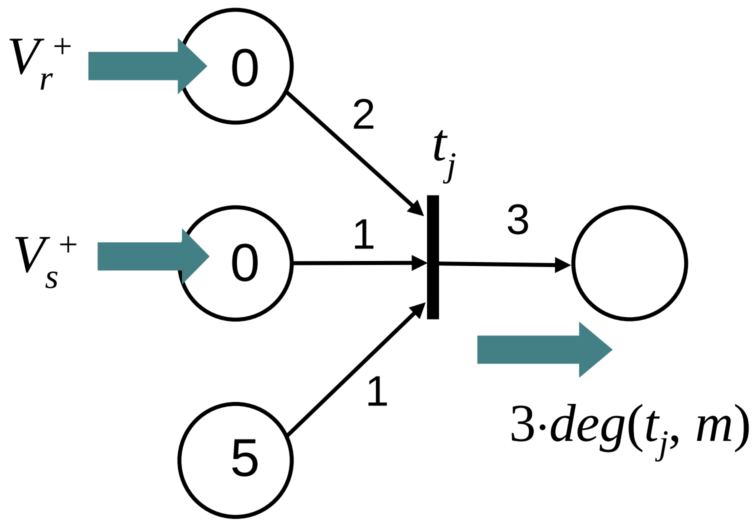


TCPN model

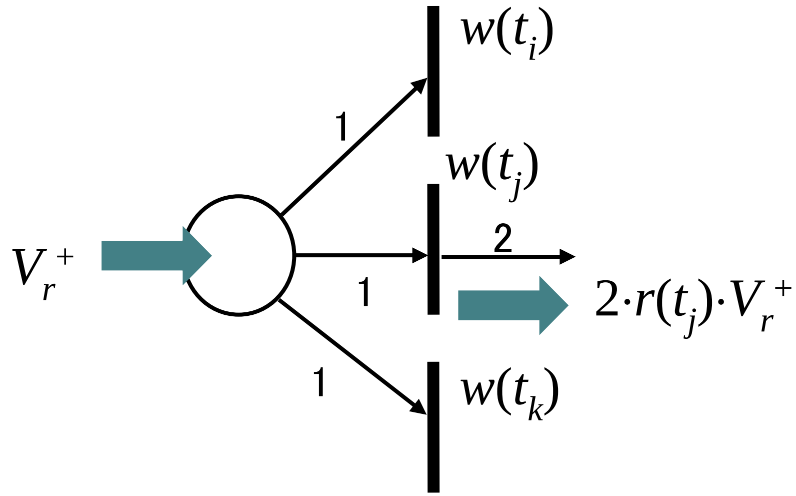


x_p
N
accept$_2$
x_{a2}
$[\lambda_{a2}]$
conditional
w_{a2}
$[\lambda_{r2}]$
w_{ca}
reject$_2$
x_{ca}
$[\lambda_{ca}]$
submit
w_{a1}
w_{r2}
x_{r2}
1
λ_s
x_s
start
accept$_1$
x_{a1}
$[\lambda_{a1}]$
end
w_{r1}
reject$_1$
x_{r1}
$[\lambda_{r1}]$
Timed transition
Routing transition

Routing transitions



$$deg(t_j, m) = min\{ V_r^+ / 2, V_s^+ \}$$



$$r(t_j) = w(t_j) / (w(t_i) + w(t_j) + w(t_k))$$

Representation by a PWL system

$$
\begin{aligned}
\dot{x}_s &= \lambda_s - R(x) \\
\dot{x}_p &= 0 \\
\dot{x}_{ca} &= \rho_{ca} R(x) - [\lambda_{ca}] x_{ca} \\
\dot{x}_{a1} &= \rho_{a1} R(x) - [\lambda_{a1}] x_{a1} \\
\dot{x}_{r1} &= \rho_{r1} R(x) - [\lambda_{r1}] x_{r1} \\
\dot{x}_{a2} &= \rho_{a2} [\lambda_{ca}] x_{ca} - [\lambda_{a2}] x_{a2} \\
\dot{x}_{r2} &= \rho_{r2} [\lambda_{ca}] x_{ca} - [\lambda_{r2}] x_{r2} \\
x_p &= 0, x_s > 0
\end{aligned}
$$

$$\xrightarrow{x_s = 0} \qquad \xleftarrow[x_p = 0]{}$$

$$
\begin{aligned}
\dot{x}_s &= 0 \\
\dot{x}_p &= R(x) - \lambda_s \\
\dot{x}_{ca} &= \rho_{ca} \lambda_s - [\lambda_{ca}] x_{ca} \\
\dot{x}_{a1} &= \rho_{a1} \lambda_s - [\lambda_{a1}] x_{a1} \\
\dot{x}_{r1} &= \rho_{r1} \lambda_s - [\lambda_{r1}] x_{r1} \\
\dot{x}_{a2} &= \rho_{a2} [\lambda_{ca}] x_{ca} - [\lambda_{a2}] x_{a2} \\
\dot{x}_{r2} &= \rho_{r2} [\lambda_{ca}] x_{ca} - [\lambda_{r2}] x_{r2} \\
x_p &> 0, x_s = 0
\end{aligned}
$$

$$R(x) = [\lambda_{a1}] x_{a1} + [\lambda_{r1}] x_{r1} + [\lambda_{a2}] x_{a2} + [\lambda_{r2}] x_{r2}$$

$$\rho_j = w_j / (w_{ca} + w_{a1} + w_{r1}) \; (j \in \{ca, a1, r1\})$$

$$\rho_j = w_j / (w_{a2} + w_{r2}) \; (j \in \{a2, r2\})$$

Interval method (1)

Ordinary differential equation.

$$\dot{x} = f(x,\theta),\ x(0) = x_0$$

θ: the vector of interval system parameters, $\theta \in [\theta]$.

Taylor series expansion.

$$x(t_{k+1}) = x(t_k) + \sum_{r=1}^{\gamma} \frac{h^r}{r!} f^{(r-1)}(x(t_k),\theta) + e(x(\eta),\theta) \quad (h = t_{k+1} - t_k,\ t_k \le \eta \le t_{k+1})$$

(γ = 1 is used in the experiments since the system is piecewise linear.)

Estimation of discretizing error by Bounding Box.

$$e(x(\eta),\theta) \subseteq [e_k] = \frac{h^{\gamma+1}}{(\gamma+1)!} F^{(\gamma)}([B_k],[\theta]), \quad [B_k]: \forall t_k \le t \le t_{k+1}.\ x(t) \in [B_k]$$

Interval method (2)

Computation of Bounding Box: iteration of Picard operator.

$$\Phi([B_k]) := [x_k] + [0, h] \cdot F([B_k], [\theta]) \subseteq [B_k]$$

Piecewise interval method:
intervals are computed by solving optimization problems.

$$[x(t_{k+1})]_i = \left[\inf_{x \in [x(t_k)] \cap \Theta, \theta \in [\theta]} \varphi_i(x, \theta), \sup_{x \in [x(t_k)] \cap \Theta, \theta \in [\theta]} \varphi_i(x, \theta) \right] + [e_k]_i$$

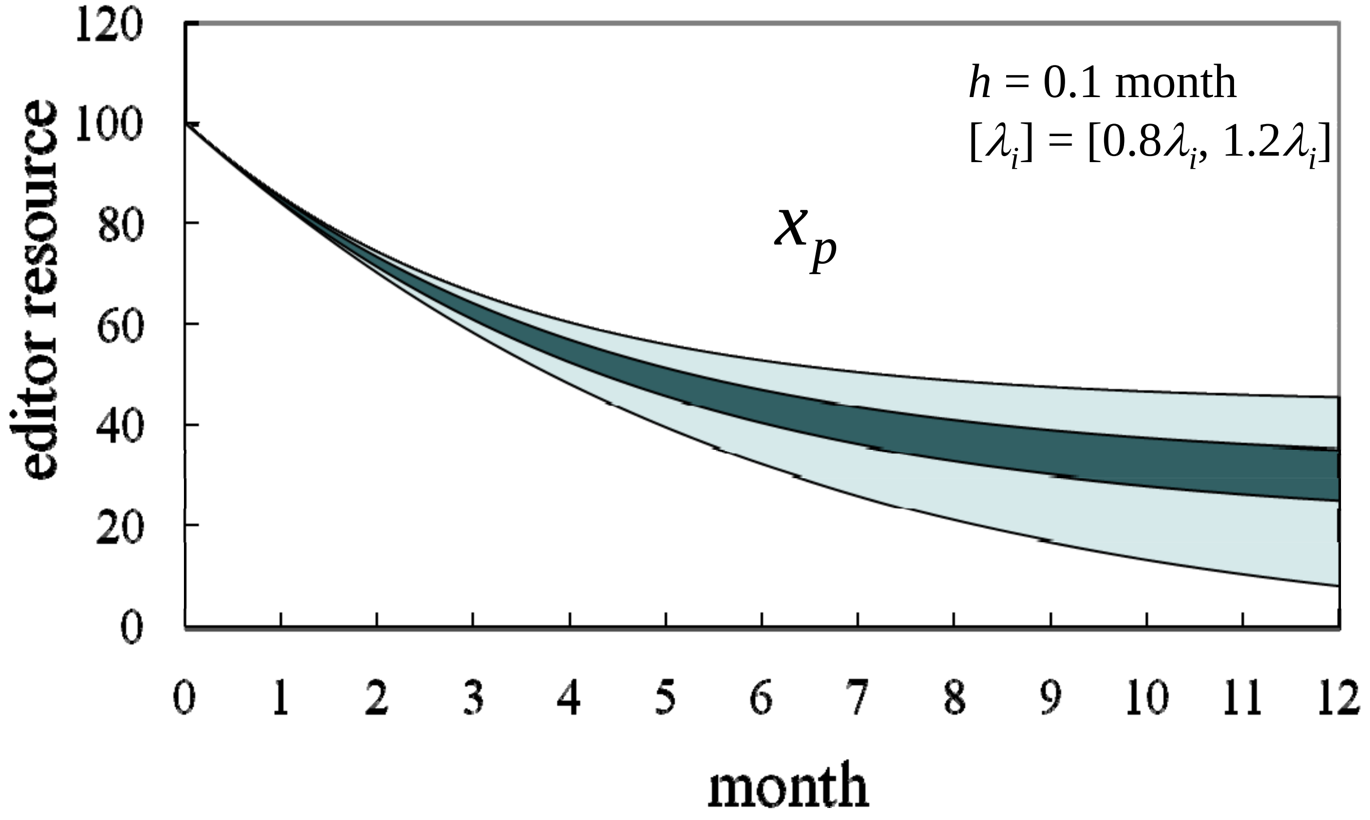
$$\varphi_i(x, \theta) := (x + \Sigma_{r=1,t} (h^r/r!) f^{(r-1)}(x, \theta)) |_i$$

Θ : constraints on the state space $\rightarrow$ we use place invariants.

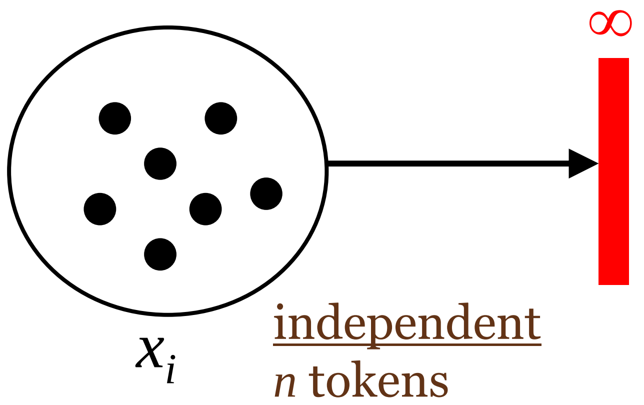
Implementation

- KCLP-HS =Prolog Interpreter
  + Linear Constraint Solver
  + Quadratic Programming Solver
  + Manipulation of Convex Polyhedra
  + Interval Arithmetic

Computation results (1)



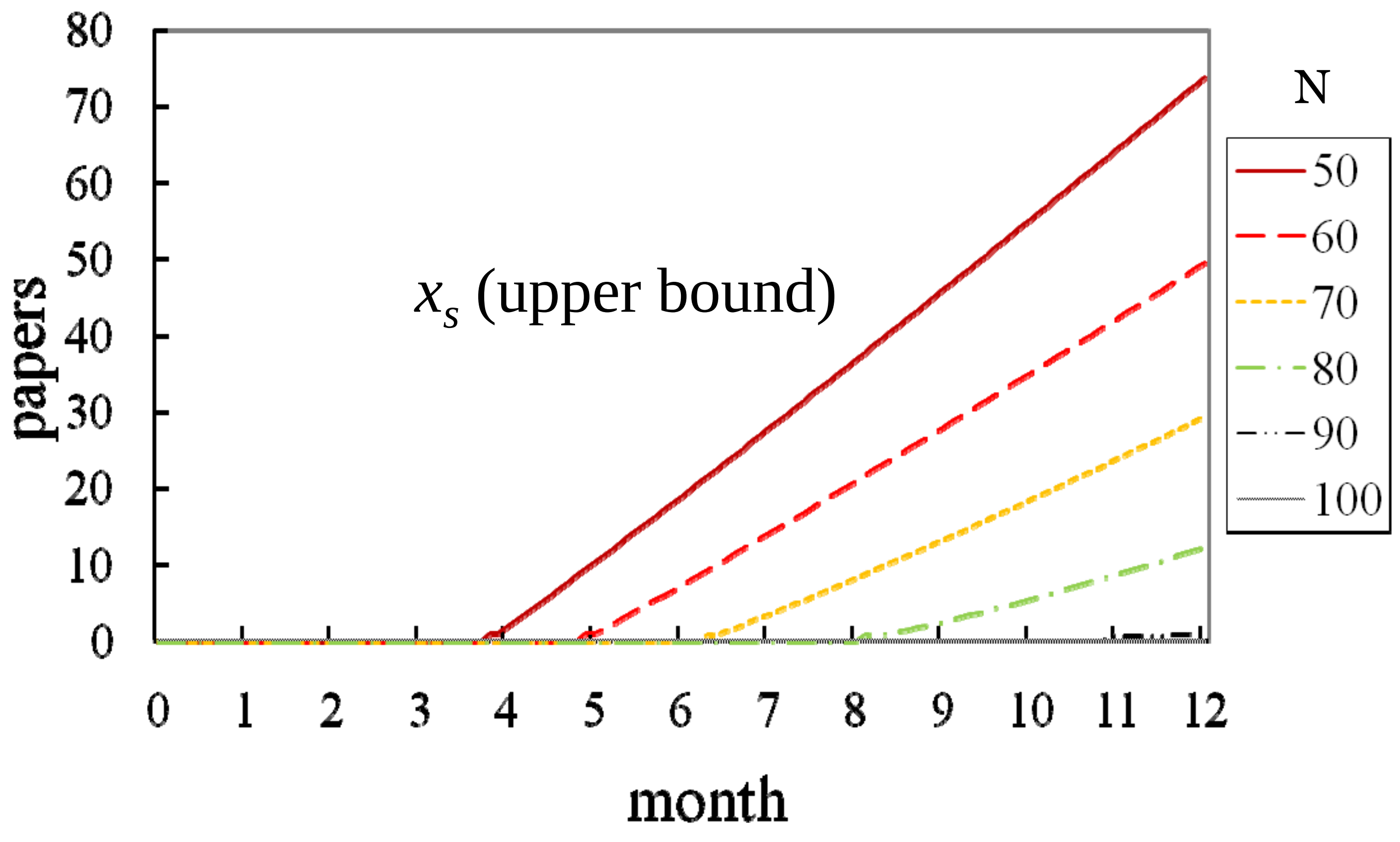
Central limit theorem



Distribution of the delay of each token: (any) identical distribution with mean τ_i, variance σ^2	→	Distribution of *the mean delay of n tokens*: normal distribution with mean τ_i, variance σ^2/n.

The interval $[\lambda]$ for each firing speed may be narrowed to $[\lambda]/\sqrt{n}$.

Computation results (2)



Conculusion

- Proposed approach
  - Computation based on linear calculation.
  - Scalable for the number of workflow instances.
  - Guaranteed enclosures for the state vector.
- Future work
  - Application to real-time systems with interval timing constraints.
  - Computation of transient behavior = Bounded Model Checking.