Bose–Einstein Condensation and Superfluidity

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Preface

Bose–Einstein condensation was discovered in atomic gas systems, where Bose condensate occupies 100% of the total system at zero temperature. Liquid helium systems have been investigated based on the Landau theory, where the superfluid component of liquid helium is background flow. According to the Landau theory, it is doubtful that the superfluid component is a Bose condensate.

In experiments, the probability of helium atoms with zero momentum is a few percent of the total liquid helium at ultra-low temperatures. However, the superfluid component occupies 100% of the liquid helium at zero temperature, as macroscopic observations indicate. These two properties of liquid helium mean that the set of helium atoms with zero momentum is not a good approximation of the ground state. What state represents the superfluid component of liquid helium?

We introduce a quasi-particle representing an eigenstate of the total Hamiltonian. We designate the quasi-particle a “dressed boson”. It is the most straightforward answer to the question posed above: the superfluid component is a Bose condensate of dressed bosons.

Experimental data of thermodynamic quantities differ greatly from the theoretical values of the Landau theory near the λ point. The specific heat has a logarithmic singularity at the λ point in the experimental data; however, the theoretical result of the Landau theory has no singularity.

In the present article, the diagonalized form of the total Hamiltonian is examined and is clarified to have a nonlinear form for the distribution function of the dressed bosons. The nonlinear form produces logarithmic divergence of the specific heat.

Many theoretical approaches have used a linear form in a total energy of a Bose system as

$$E = \sum_i \varepsilon_i n_i \ ,$$

where n_i is the quasi-particle number in quantum level *i*, and where ε_i is the energy per quasi-particle. This familiar form maintains the order of energy from small to large. That is to say, the energy of level 1 is smaller than that of level 2 always if $\varepsilon_1 < \varepsilon_2$. The property changes drastically for a nonlinear form of a total energy as

$$E = \sum_i n_i \varepsilon_i + \sum_{i,j} f_{ij} n_i n_j + \cdots .$$

The energy of a quasi-particle (dressed boson) is definable as

$$\frac{\partial E}{\partial n_i} = \varepsilon_i + \sum_j f_{ij} n_j + \sum_j f_{ji} n_j + \cdots ,$$

which depends upon the other dressed boson numbers. Consequently, the energy of the dressed boson with quantum level *i* varies depending on the distribution of dressed boson number. This nonlinear dependence yields level inversion; that is to say, which momentum level of the dressed boson has a minimum energy depends upon the choice of the distribution of dressed boson number. The level with momentum zero has minimum energy for some distribution. However, when the distribution of dressed boson number changes into a specific distribution, a level with a non-zero momentum has minimum energy. This level inversion produces Bose condensation of the dressed bosons with non-zero momentum. The stability of the moving superfluid component is established on the basis of this level inversion. Many other surprising effects arise from the nonlinearity.

In almost all cases for many body problems, the total energy is nonlinearly dependent upon the distribution function of quasi-particle number. Accordingly, the developed method explained in this book is widely applicable to investigation of the statistical physics of many body problems.

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